

## Comparative efficiency of various Mathematical functions on unadjusted lactation curve in Jaffrabadi Buffalo

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### Abstract

A study was carried out on 310 normal lactations (1-6 parity) pertaining to 123 Jaffrabadi buffaloes spread over 15 years (1978-1992) maintained at Cattle Breeding Farm, Junagadh Agricultural University, Junagadh (Gujarat). Four lactation curve models viz. Exponential, Parabolic exponential, Inverse polynomial and Gamma-type functions were fitted on weekly milk yields and their relative efficiency was compared. Gamma-type function described the best fit with highest coefficient of determination (99.82%) whereas, least coefficient of determination (85.25%) was observed in Exponential function. Exponential function could not explain initial and peak phase of the lactation, however, it explains only the descending segment of the lactation curve. The trend of Inverse polynomial function, however, indicated that the function might be most suited to buffaloes which start their lactation at very low level of production, reach their peak very early and then start declining at an early stage. Gamma-type and Inverse polynomial functions should be preferred over others for predicting milk yield at any stage of lactation in Jaffrabadi buffalo. Gamma-type function is recommended for lactation data of Jaffrabadi buffalo to achieve efficient breeding and feeding management of dairy herds.

**Keywords:** Jaffrabadi buffalo, lactation curve, milk yield

### Introduction

Graphical representation of milk production throughout the lactation period forms the lactation curve. In dairy buffaloes milk production follow a curvilinear pattern over the course of lactation. Knowledge of the lactation curve can provide a worthwhile information source about the pattern of milk production traits which can be used for designing suitable breeding and management strategies and prediction of total milk yield for dairy buffaloes. Various models have been tried by different researchers to fit the

lactation curve in cattle and buffaloes. However little work is available in Jaffrabadi buffaloes, which is considered to be one of the best milk breed of dairy buffaloes in India. Therefore, our objective was to choose the most appropriate function which is more close to the observed distribution of milk yield with reasonably high degree of predictability in Jaffrabadi buffalo.

Rajendra Kumar and Bhat (1978) established the lactation curve function based on

individual lactation of Indian buffaloes. They concluded that the use of inverse polynomial and gamma-type functions should be preferred to others while studying the individual lactation curve in Indian buffaloes and to forecast the milk yield at any stage of lactation. Bhat and Rajendra Kumar (1980) and Cheena and Basu (1983) noticed that Gamma-type function followed by Inverse polynomial function were more close to the observed lactation curve in Murrah buffalo. Kumar *et al.* (1993) concluded that gamma-type function had the best fit in explaining the lactation curve of early lactation in Murrah buffaloes. Dongre *et al.* (2013) reported that inverse polynomial function described the best fit with highest coefficient of determination (99.92 %) on fortnightly test day milk yield in Sahiwal cows.

### Materials and methods

A study was carried out on 310 normal lactations (1-6 parity) pertaining to 123 Jaffrabadi buffaloes spread over 15 years (1978-1992) maintained at Cattle Breeding Farm, Junagadh Agricultural University, Junagadh (Gujarat). Four lactation curve models viz. Exponential, Parabolic exponential, Inverse polynomial and Gamma-type functions were fitted on weekly milk yield data up to 45<sup>th</sup> week of lactations and their relative efficiency was compared.

### Lactation Curve Models

Following four mathematical models were fitted to find out the best fit curve

#### 1. Exponential function: (Brody *et al.*, 1923)

$$Y_t = Ae^{-ct}$$

Where,

$Y_t$  = is the average milk yield at time 't'

A = is the initial milk yield

c = is rate of change of decline

#### 2. Parabolic Exponential function: (Sikka, 1950)

$$Y_t = Ae^{(bt-ct^2)}$$

Where,

$Y_t$  = is the average daily milk yield at the time 't'

A = is the initial milk yield

b = is the linear constant which measures the average slope of the curve

c = describes how on an average the rate of slope varied from period to period

#### 3. Gamma type function: (Wood, 1967)

$$Y_t = At^b \cdot e^{-ct}$$

Where,

$Y_t$  = is the average daily milk yield at the time 't'

A = is the initial milk yield

b = is the average slope of the curve

c = describes rate of change for decline phase

#### 4. Inverse polynomial function: (Nelder, 1966)

$$Y_t = t[A + bt + ct^2]^{-1}$$

Where,

$Y_t$  = is the average daily milk yield at the time 't'

A = is the raising extremes of the curve

b = is the constant phase of the curve

c = is the declining extremes of the curve

### Results and discussion

The present investigation was undertaken to establish a lactation curve function in Jaffrabadi buffalo. Four appropriate functions viz. Exponential (Brody, *et al.*, 1923, Parabolic exponential (Sikka, 1950), Inverse polynomial (Nelder, 1966) and Gamma-type function (wood, 1967) were compared. A model situation in which the functions under estimation were plotted against an unadjusted overall (observed) lactation curve (Figure 1).

Following equations of different curve functions were estimated.

1. Exponential function:

$$Y_t = 66.4268 e^{-0.0203t}$$

2. Parabolic Exponential function:

$$Y_t = 53.6564 e^{0.0069t - 0.00059t^2}$$

3. Gamma type function:

$$Y_t = 42.6062t^{0.3037} \cdot e^{-0.0389t}$$

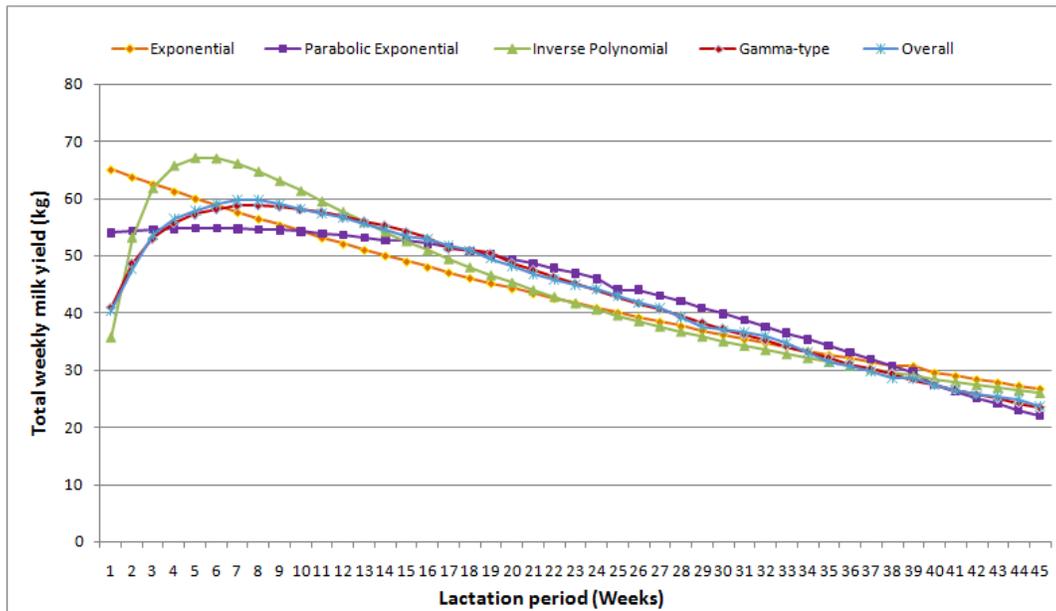
4. Inverse polynomial function:

$$Y_t = t[0.0824 + 0.0027t + 0.00092t^2]^{-1}$$

The most suitable model was identified on the basis of the highest value of coefficient of determination ( $R^2$ ). The  $R^2$  values for Exponential, Parabolic exponential, Inverse polynomial and Gamma-type functions accounted for 85.25, 95.00, 99.76 and 99.82 % of the variation respectively. Maximum  $R^2$  value was observed for Gamma-type function (99.82%) followed by Inverse polynomial (99.76%), Parabolic exponential (95.00%) and Exponential (85.25%) functions. The difference in  $R^2$  value

between Gamma-type and Inverse polynomial functions was very small. The results were in agreement with those reported by Rajendra Kumar and Bhat (1979), Bhat and Rajendra Kumar (1980) and Cheema and Basu (1983) in Indian buffaloes and Kumar *et al* (1993) in Indian buffaloes having been calved before 42 months of age.

Exponential curve indicated that milk yield was linearly related with advancement of lactation period during the descending phase of lactation curve. Therefore, it can neither describe the initial increase of milk nor peak production. It can only describe the declining phase. Consequently, it cannot figure in any discussion of lactation curve in its totality. It can be concluded, therefore, that exponential curve has merit to represent only declining phase of lactation curve in Jaffrabadi buffalo. The graph (Figure 1) ran close to the observed values only from 23<sup>rd</sup> week onward. So this function can be used to estimate milk yield only of the declining phase of the lactation. The result in conformity with that of Rajendra Kumar and Bhat (1979) in Indian buffaloes.



**Fig. 1: The overall (observed) and predicted weekly milk yield from different lactation curve functions in Jaffrabadi buffalo.**

**Appendix I: Observed and predicted values of weekly milk yield by different lactation curve functions in Jaffrabadi buffalo.**

Week No	Predicted total weekly milk yield (kg)				Observed total weekly Milk yield (kg)
	Exponential	Parabolic Exponential	Inverse Polynomial	Gamma-type	Overall (Observed)
1	65.09	53.99	35.79	40.98	40.32
2	63.78	54.28	53.19	48.6	47.74
3	62.49	54.49	61.76	52.95	53.56
4	61.24	54.64	65.68	55.62	56.38
5	60	54.73	67.04	57.19	57.78
6	58.79	54.75	66.96	58.1	59.02
7	57.59	54.71	66.05	58.66	59.73
8	56.45	54.6	64.67	58.69	59.65
9	55.32	54.43	63.03	58.55	58.94
10	54.21	54.19	61.27	58.06	58.16
11	53.11	53.89	59.46	57.56	57.29
12	52.04	53.54	57.66	56.72	56.66
13	50.99	53.12	55.89	55.9	55.58
14	49.97	52.64	54.17	55.19	54.64
15	48.96	52.64	52.52	54.13	53.41
16	47.98	52.09	50.93	53.02	52.85
17	47.02	51.51	49.42	51.16	51.69
18	46.07	50.86	47.97	50.87	50.92
19	45.14	50.16	46.59	50.43	49.48
20	44.23	49.41	45.27	48.66	48.16
21	43.34	48.62	44.02	47.37	46.74
22	42.47	47.81	42.83	46.26	45.86
23	41.62	46.9	41.69	45.18	44.94
24	40.78	45.99	40.61	43.99	44.16
25	39.96	44.04	39.58	42.82	42.98
26	39.15	44.05	38.59	41.69	41.74
27	38.37	43.04	37.65	40.59	40.9
28	37.59	41.99	36.75	39.4	39.21
29	36.84	40.93	35.9	38.36	37.73
30	36.09	39.85	35.07	37.23	37.03
31	35.37	38.75	34.29	36.14	36.73
32	34.66	37.63	33.53	35.18	35.99
33	33.96	36.5	32.81	34.15	34.75
34	33.27	35.37	32.12	33.15	33.1
35	32.61	34.23	31.45	32.18	31.59
36	31.95	33.09	30.81	31.14	30.75
37	31.31	31.94	30.2	30.22	29.86
38	30.68	30.81	29.61	29.34	28.66
39	30.6	29.67	29.04	28.29	28.57
40	29.45	27.43	28.49	27.55	27.38
41	28.86	26.32	27.96	26.67	26.54
42	28.28	25.24	27.45	25.88	25.79
43	27.71	24.16	26.96	25.05	25.42
44	27.15	23.11	26.49	24.24	24.94
45	26.61	22.07	26.03	23.52	23.73

Inverse polynomial function indicated that, in the initial stage of lactation, the expected milk yield was close to the observed milk yield up to 3<sup>rd</sup> week and then overestimated it from 3<sup>rd</sup> to 12<sup>th</sup> week. Thereafter, the curve ran close to the average observed lactation curve. Inverse polynomial function showed very low initial (First week) milk yield and higher peak yield, giving rise to a steep ascending phase compare to other functions. The trend of the inverse polynomial function, however, indicated that the function might be most suited to buffaloes which start their lactation at very low level of production, reach their peak very early and then start declining at an early stage. Similar results have been reported by Rajendra Kumar and Bhat (1979) in Indian buffaloes and Cheena and Basu (1983) in Murrah buffaloes.

Parabolic exponential function indicated wide diversity between the actual and expected yield up to 17<sup>th</sup> week of production and thereafter the values were closer. The result is in agreement with those reported by Kumar *et al.* (1993) for the Indian buffaloes calved after 42 months of age.

Gamma-type curve gave a very close fit with the observed lactation curve with highest  $R^2$  value (99.82 %). In the ascending phase, the expected production was very close to the observe up to 4<sup>th</sup> week, the expected yield was slightly lower than the observed from 5<sup>th</sup> to 10<sup>th</sup> week and thereafter it was close to the observed value. The peak production in the expected curve was observed in the 8<sup>th</sup> week while in the observed curve it was in the 9<sup>th</sup> week. It gave close fit in the ascending phase of the curve. Similar results have been reported by Rajendra Kumar and Bhat (1979), Bhat and Rajendra Kumar (1980) in Indian buffaloes, Cheena and Basu (1983) in Murrah buffaloes and Kumar *et al.* (1993) in Indian buffaloes having been calved before 42 months age.

Considering the graphical comparison of the above models and near  $R^2$  values of Inverse polynomial and Gamma-type functions with observed lactation curve, Gamma-type function should be preferred to others for lactation data of Jaffrabadi buffaloes. The precise fitting of Gamma-type lactation curve will help dairy producers use records to achieve efficient breeding and feeding management of dairy herds.

### Conclusion

Gamma-type and Inverse polynomial functions should be preferred over others for predicting milk yield at any stage of lactation in Jaffrabadi buffalo. Gamma-type function is recommended for lactation data of Jaffrabadi buffalo.

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