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Optimal Control and Bifurcation Study of Habitat Destruction Model with Three-Species

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Abstract

This paper is devoted to study the problem of optimal control of habitat destruction model with prey-predator -top predator and the bifurcation behavior of the system. Using the Lyapunov technique and Krasovsikii theorem, the optimal control functions ensuring asymptotic stability of system steady states are obtained and found to be as functions of the phase state and time. As an application, it was shown that the equilibrium states of the habitat destruction model with prey-predator -top predator system are asymptotic stable. Bifurcation phenomena of the system at the trivial solution is studied and presented graphically. Numerical examples are presented graphically to illustrate the theoretical obtained results and the efficient of the used methods.

Keywords: Prey, Top- Predator, Three-species, Estimation, Habitat destruction, Lyapunov function, Bifurcation

1. Introduction

The natural and bounded coexistence between the organisms is one of the coefficients that satisfies the balance of the environment .For example, Prakash and de Ross [1] have found that the presence of predators is useful for preys existence.

The increasing of the habitat destruction is causing an imbalance in the densities of the populations of prey and predator. in other hand the accumulation of the destruction in a local area increases the risk of extinction in a bigger area and. So, we can say that, the causal relation between species extinction and the habitat destruction is very complicated [2, 3, 4].

The change in the size of objects density, increasing or decreasing has many causes

The variation of the total population size, increasing or decreasing has many causes, for example, it is determined by the birth and the mortality rates of species [5]. Also, the increasing in the size of a type of objects, may be means a lack in the size of other.

Recent models of competition indicate that, the effect of habitat destruction on coexisting between preys and predators is dependent on the ratio of extinction risk due to predation and prey colonization rate [6], [7].

The environmental pollution is an important topic and is especially appreciated by the developed countries, which conduct many studies and conferences in this regard, which are discussed topics related to the issues of challenges to economic development and human rights as a result of environmental pollution, such as the conference entitled "The effects of climate change on global economy and human health" December - 2017 in Dallas Texas USA , sponsored by Environmental Protection and Human Rights Organization.

This paper is an extension of the study [8], which has studied the stability and estimation of the unknown parameter of the system. It has found that the system has a high chaotic behavior. Also, the controller laws are found as non-linear functions of the species densities, and the dynamic estimators of the unknown parameters and its updating rules over time are derived from the conditions of the asymptotic stability of the system around its steady states.

In this paper we will try to achieve the optimal behavior of the chaotic model using the Lyapunov function and Krasovskii theorem. The paper [8] and this study also, are considered as a complement for the results of the study [9], that is concentrated on two-species model, (prey and predator) which found that the system, in general is unstable and has a chaotic behavior. More details are accessible through going back to searches. Alwan also, has studied the stability and behavior for the model of stochastic lattice gas of prey-predator model with pair-approximation. She found that this system has a chaos behavior and she has derived the estimators of the unknown parameters and the updating roles that satisfied the asymptotic stability behavior [10].

Usually, the dynamical system that contains of parameters, the values of these parameters are often only known approximately. Sometimes, it can happen a slight variation in a parameter can have significant impact on the solution. Thus it is important to study the behavior of solutions and examine if there is a qualitative change in the behavior of the dynamical system as some parameters are change. One of these changes in the behavior of the dynamical systems when changing the values of its parameters is called bifurcation. Al-Mahdi and Khirallah have studied stability and bifurcation analysis of a model of cancer in [11], [12].

This paper has the following structure. In Section 2, the lattice model and its assumed interactions , the mathematical formula of the model and its stationary states will be presented. Section 3 is devoted to study the optimal control problem and deriving the optimal controllers functions using the Lyapunov function and Krasovskii theorem. In Section 4, Lyapunov asymptotic stability will be discussed. In Section 5, the bifurcation analysis of the system. Section 6 is dedicated to the numerical solutions and graphical presentation. Finally, Conclusions are provided in Section 7.

2. The Lattice Model

In this section, the assumed interactions of the proposed model will be discussed and the mathematical formula will be presented.

Each site in the lattice can be labeled by X, Y, Z, or O, where X $(or\ Y, Z)$ is the site occupied by prey (or predators), and O represents the vacant site. The assumed interactions of this model are given by [2]

$$\left.\begin{array}{llll} Y+Z \xrightarrow{1} 2Z & (1.1) & X \xrightarrow{r1} O & (1.4) \\ X+Y \xrightarrow{1} 2Y & (1.2) & Y \xrightarrow{r2} O & (1.5) \\ X+O \xrightarrow{r0} 2X & (1.3) & Z \xrightarrow{r3} O & (1.6) \end{array}\right\} \quad (1)$$

The above interactions respectively represent two kinds of predation with probability 1, reproduction of prey with probability $r0$ and the deaths of the three-species prey, predator and top-predator, with probabilities $r1$, $r2$, $r3$ respectively. In the lattice boundary, the barriers are putting as a link between neighboring sites with probability p which called barrier density. The probability p can be used as a measures of the intensity of habitat destruction, because the different values of p from zero to one generate different and important situations. More details about these situations can be seen in [1].

2.1 The mathematical formula of the model

The three-species model in mathematical form is given by

$$\dot{w} = -2r_0(1-p)wx + r_1 x + r_2 y + r_3 z \quad (2.1)$$

$$\dot{x} = 2r_0(1-p)wx - r_1 x - 2\,x\,y \quad (2.2)$$

$$\dot{y} = 2\,x\,y - 2\,y\,z - r_2\,y \quad (2.3)$$

$$\dot{z} = 2\,y\,z - r_3\,z \quad (2.4)$$

where the dot represents the derivative with respect to time, w , x, y and z are the densities $of\ O, X, Y\ and\ Z$ respectively, ri , i = 0, 1, 2, 3 are four unknown probabilities, which represent the probabilities of reproduction of prey, deaths of the prey, predator and top-predator, respectively, and p is *barrier density*. The state variables of this system are subject to the relationship

$w + x + y + z = 1$ (3)

Therefore, for simplicity in our study, one can take just three equations instead of four, which are considered sufficient to study the problem of optimal control of the system. Accordingly, using the relation (3), one can reduce the model (2) to be as the following form:

$$\dot{x} = 2r_0(1-p)(1-x-y-z)x - r_1 x - 2\,x\,y \quad (4.1)$$

$$\dot{y} = 2\,x\,y - 2\,y\,z - r_2\,y \quad (4.2)$$

$$\dot{z} = 2\,y\,z - r_3\,z \quad (4.3)$$

3. Optimal Control Functions

In this section we will study the problem of optimal control of the habitat destruction model with prey, predator and top-predator using the Lyapunov technique and Krasovsikii theorem [13]. To apply this technique let us introduced three external inputs which will be designed to achieve the optimal behavior of the system to its steady states. Thus, the dynamical equations of the model can be written as the following form

$$\dot{x} = 2r_0(1-p)(1-x-y-z)x - r_1 x - 2xy + V_1 \quad (5.1)$$

$$\dot{y} = 2xy - 2yz - r_2 y + V_2 \quad (5.2)$$

$$\dot{z} = 2yz - r_3 z + V_3 \quad (5.3)$$

where Vi, i = 1,2,3 are three introduced control inputs. The controlled system is given by solving the following non-linear system

$$2r_0(1-p)(1-x_0-y_0-z_0)x_0 - r_1 x_0 - 2x_0 y_0 + V_{10} = 0 \quad (6.1)$$

$$2x_0 y_0 - 2y_0 z_0 - r_2 y_0 + V_{20} = 0 \quad (6.2)$$

$$2y_0 z_0 - r_3 z_0 + V_{30} = 0 \quad (6.3)$$

where Vi0 and x0, y0, z0 are the values of the control functions and the state variables at the system equilibrium states, where the equilibrium states of the system (4) are given by [8]

$$S_1 = (0,0,0) \quad (7.1)$$

$$S_2 = \left(0, \frac{r_3}{2}, -\frac{r_2}{2}\right) \quad (7.2)$$

$$S_3 = \left(\frac{\alpha - r_1}{\alpha},\ 0,\ \ 0\right) \quad (7.3)$$

$$S_4 = \left(\frac{r_2}{2}, \alpha(1-r_2/2)/(2+\alpha),\ \ 0\right) \quad (7.4)$$

$$S_5 = \left(\frac{1}{2} + \frac{r_2 - r_3}{4} - \frac{r_1 + r_3}{2\alpha}, \quad \frac{r_3}{2}, \quad \frac{1}{2} - \frac{r_2 + r_3}{4} - \frac{r_1 + r_3}{2\alpha}\right) \qquad (7.5)$$

where,

$$\alpha = 2r_0(1 - p) \geq 0 \quad (8)$$

The first stationary state $S1$ corresponds to *a vacuum-absorbing* state, the second stationary state $S2$ is absolutely biologically inadmissible because the density of the top-predator is negative, while the necessary conditions to be $S3$, $S4$ and $S5$ biologically admissible are: $\alpha \geq r1$, $\alpha \geq 2r1/(2 - 2r1)$ and $\alpha \geq 2(r1 + r3)/(2 - r2 - r3)$ respectively. Reference [8] found that the habitat destruction model with three species (prey, predator and top-predator) is unstable. Some graphical examples illustrated that in Section (6).

To obtain the control functions Vi , we will introduce the following new variables

$$\xi_1 = x - x_0 \qquad (9.1)$$

$$\xi_2 = y - y_0 \qquad (9.2)$$

$$\xi_3 = z - z_0 \qquad (9.3)$$

$$U_i = V_i - V_{i0}, i = 1,2,3 \qquad (9.4)$$

Thus $\dot{x} = \xi_1, \ \dot{y} = \xi_2, \ and \ \dot{z} = \xi_3,$ where the dot refers to the derivation with respect to time. Substituting Eq. (9) into (5), we get the following system

$$\dot{\xi}_1 = -\alpha(\xi_1 + \xi_2 + \xi_3)(\xi_1 + x_0) + \alpha(1 - x_0 - y_0 - z_0)\xi_1 - 2\xi_1\xi_2 - 2y_0\xi_1$$
$$-2x_0\xi_2 - r_1\xi_1 + U_1 \qquad (10.1)$$

$$\dot{\xi}_2 = 2\xi_1\xi_2 + 2y_0\xi_1 + 2x_0\xi_2 - 2\xi_2\xi_3 - 2z_0\xi_2 - 2y_0\xi_3 - r_2\xi_2 + U_2 \qquad (10.2)$$

$$\dot{\xi}_3 = 2\xi_2\xi_3 + 2z_0\xi_2 + 2y_0\xi_3 - r_3\xi_3 + U_3 \qquad (10.3)$$

The optimal control functions Ui will determine from the conditions that ensure the asymptotic stable of the equilibrium states (7). We proceed to obtain the stabilizing control functions Vi as functions of ξ1, ξ2 and ξ3. To do that we define a new control function Li , i = 1,2,3 as the following

$$L_1 = -\alpha\xi_1^2 - \alpha(\xi_1 + \xi_2 + \xi_3)x_0 + \alpha(1 - x_0 - y_0 - z_0)\xi_1 - 2y_0\xi_1 - 2x_0\xi_2$$
$$-r_1\xi_1 + 2\xi_2^2 + U_1 \qquad (11.1)$$

$$L_2 = 2y_0\xi_1 + 2x_0\xi_2 - 2z_0\xi_2 - 2y_0\xi_3 - r_2\xi_2 - (2 + \alpha)\xi_1^2 + U_2 \qquad (11.2)$$

$$L_3 = 2z_0\xi_2 + 2y_0\xi_3 - r_3\xi_3 - 2\xi_2^2 - \alpha\xi_1^2 + 2\xi_2\xi_3 + U_3 \qquad (11.3)$$

Then,

$$\dot{\xi}_1 = L_1 - (2 + \alpha)\xi_1\xi_2 - \alpha\xi_1\xi_3 - 2\xi_2^2 \qquad (12.1)$$

$$\dot{\xi}_2 = L_2 + 2\xi_1\xi_2 - 2\xi_2\xi_3 + (2 + \alpha)\xi_1^2 \qquad (12.2)$$

$$\dot{\xi}_3 = L_3 + 2\xi_2^2 + \alpha\xi_1^2 \qquad (12.3)$$

Now, we will obtained the external controls Vi which are functions of ξi , i = 1,2,3, such that, the trivial solution of the system (10) will asymptotic stable and satisfy minimization of the following performance measure

$$F = \int_{t0}^{\infty} \left\{ \sum_{i=1}^{3} k_i L_i^2 + l_i \xi_i^2 \right\} dt, \qquad (13)$$

where t_0 is a fixed time can be choose from new condition that can be add to consideration, k_i , l_i are positive constants and using Krasovskii's theorem and fundamental

Lyapunov's theorem for optimal stabilization, we have

$$L_i = -\frac{1}{2k_i}\frac{\partial\varphi}{\partial\xi_i} \quad (14)$$

and

$$\frac{\partial\varphi}{\partial\xi_1}\left(-(2+\alpha)\xi_1\xi_2-\alpha\xi_1\xi_3-2\xi_2^2\right)+\frac{\partial\varphi}{\partial\xi_2}\left(2\xi_1\xi_2-2\xi_2\xi_3+(2+\alpha)\xi_1^2\right)+\frac{\partial\varphi}{\partial\xi_3}\left(2\xi_2^2-\alpha\xi_1^2\right)$$
$$+\sum_{i=1}^{3} l_i\xi_i^2-\sum_{i=1}^{3} k_i\left(\frac{1}{2k_i}\frac{\partial\varphi}{\partial\xi_i}\right)^2=0 \qquad (15)$$

Equation (15) will be satisfied if we chose φ as the following quadratic form

$$\varphi = \sqrt{l_i k_i}(\xi_1^2 + \xi_2^2 + \xi_3^2) \quad (16)$$

Thus, the optimal feedback controllers Ui, i = 1,2,3 that ensuring asymptotic stability of the equilibrium states of habitat destruction model (5) in infinite time interval can be obtained in the form

$$U_1 = \left[\sqrt{l/k}+\alpha(1-2x_0-y_0-z_0-\xi_1)-2y_0-r_1\right]\xi_1+2\xi_2(\xi_2-x_0)$$
$$-\alpha(\xi_2+\xi_3)x_0 \qquad (17.1)$$

$$U_2 = [2y_0-(2+\alpha)\xi_1]\xi_1+\left[\sqrt{l/k}+2x_0-2z_0-r_2\right]\xi_2-2y_0\xi_3 \qquad (17.2)$$

$$U_3 = \alpha\xi_1^2+2\xi_2(z_0-\xi_2)+\left[\sqrt{l/k}+2y_0-r_3+2\xi_2\right]\xi_3 \qquad (17.3)$$

where $\boldsymbol{l_i} = \boldsymbol{l}$ and $\boldsymbol{k_i} = \boldsymbol{k}$.

4. Lyapunov Asymptotic Stability

In this section we will establish that according Ui , i = 1,2,3 in (17), the control unction the state in Eq's (7) is asymptotic stable in the Lyapunov sense. Substituting from (17) into (12) we get

$$\dot{\xi}_1 = -\alpha(\xi_2+\xi_3)\xi_1-2\xi_1\xi_2-2\xi_2^2-\sqrt{l/k}\,\xi_1 \qquad (18.1)$$

$$\dot{\xi}_2 = +2\xi_1\xi_2-2\xi_2\xi_3+(2+\alpha)\xi_1^2-\sqrt{l/k}\,\xi_2 \qquad (18.2)$$

$$\dot{\xi}_3 = 2\xi_2^2+\alpha\xi_1^2-\sqrt{l/k}\,\xi_3 \qquad (18.3)$$

The function φ in (16) clearly, is a positive-definite form with respect to ξi and its total derivative with respect to time, along the trajectories of the system (18) will be

$$\dot{\varphi} = -2l\ (\xi_1^2 + \xi_2^2 + \xi_3^2) \quad (19)$$

which is a negative-definite along the trajectories of the system (18). Then, the equilibrium states (7) of the system (5) are asymptotic stable in the Lyapunov sense. Since, our dynamical system that is described by Equation (4) contains the parameters $\boldsymbol{ri}$, $\boldsymbol{i}$ = **0**, **1**, **2**, **3** and $\boldsymbol{p}$, which represent the probabilities of reproduction of prey, deaths of the prey, predator and top-predator, and $\boldsymbol{p}$ is the barrier density. The values of these parameters are unknown. So, we will study the effect of the changing of values of some of these parameters on the behavior of the system in the next section.

5. Bifurcation Analysis

In this section we will confine to study the bifurcation at the trivial solution S1 = (0,0,0) as an indication of the presence of branches in the behavior of the system or not. To study the bifurcation at this point, we will find the Jacobean matrix $J(x, y, z)$ as follows

$$J = \begin{pmatrix} \alpha\,(1-x-y-z)-2y-r_1-\alpha x & -(\alpha+2)x & -\alpha x \\ 2y & 2x-r_2-2z & -2y \\ 0 & 2z & 2y-r_3 \end{pmatrix} \qquad (20)$$

where $\alpha = 2r_0(1-p)$. At S_1, the Jacobean matrix becomes

$$J_{S_1} = \begin{pmatrix} 2r_0(p-1) - r_1 & 0 & 0 \\ 0 & -r_2 & 0 \\ 0 & 0 & -r_3 \end{pmatrix} \qquad (21)$$

which gives the following eigenvalues

$$\lambda_1 = \alpha - r_1, \qquad \lambda_2 = -r_2 < 0,$$
$$\lambda_3 = -r_3 < 0. \qquad (22)$$

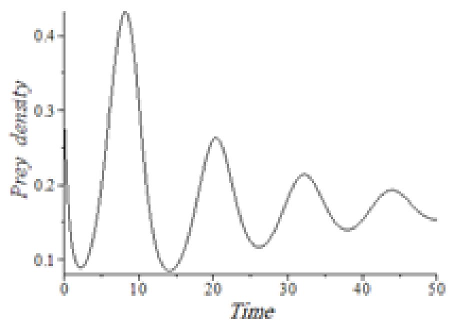
We here observe that the first eigenvalue $\lambda 1$ can be equal to zero, this means that the equilibrium state $S1 = (0,0,0)$ is a bifurcation point. If $\lambda 1 = 2r0(1 - p) - r1 = 0$, this implies that $r1 = 2r0(1 - p)$. We can see the bifurcation at the critical point $S1 = (0, 0, 0)$ with different values of the parameter $r1$ graphically in Section (6).

6. Numerical illustration

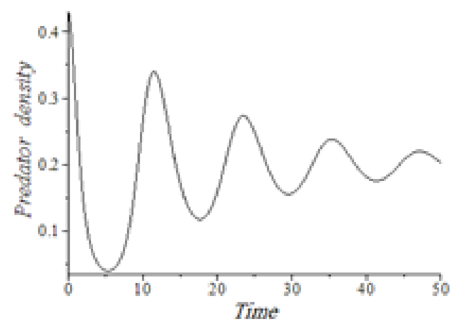
This section is devoted to present some numerical examples for the trajectory of the habitat destruction model with prey, predator and top-predator before and after adding the external optimal control parameters to illustrate the efficiency of the used method. Finally, we will illustrate the bifurcation behavior at the trivial solution.

6.1 Chaotic behavior of the system

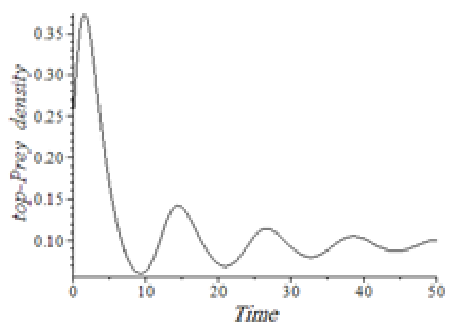
In the following figures illustrate the chaotic behavior of the uncontrolled system.



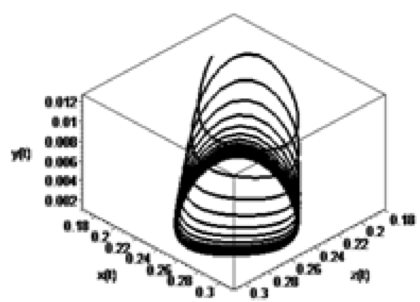
(a)



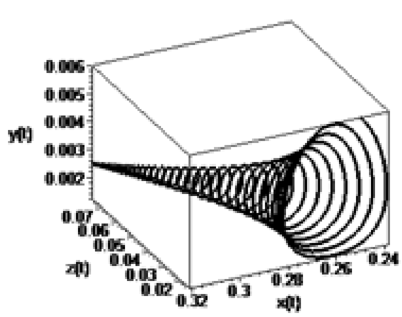
(b)



(c)



(d)



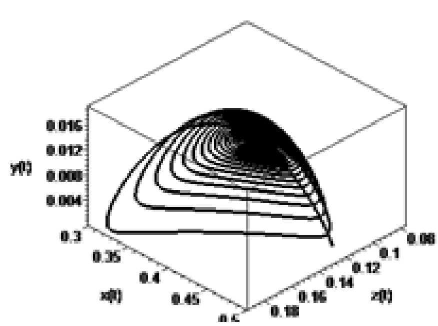


Figure (1): Chaotic behavior of the habitat destruction model with prey-predator -top predator system .

In (a), (b) and (c) the chaotic behavior of prey predator and top-predator respectively at the probabilities $r0 = 0.95$, $r1 = 0.31$, $r2 = 0.15$, $\boldsymbol{r3} = 0.4$, $p = 0.3$. In (d), (e) and (f) the attractors of the system at the probabilities ($r0 = 0.012$, $r1 = 0.0001$, $r2 = 0.007$, $\boldsymbol{r3} = 0.009$, $p = 0.12$), ($r0 = 0.012$, $r1 = 0.0001$, $r2 = 0.007$, $\boldsymbol{r3} = 0.009$, $p = 0.57$) and ($r0 = 0.01$, $r1 = 0.001$, $r2 = 0.49$, $\boldsymbol{r3} = 0.005$, $p = 0.5$) respectively.

Figure (1) (a), (b) and **(c)** show the trajectory of the uncontrolled system determined by (x, y, z) over time and the oscillating behavior of the system is clearly evident.

Figure (1) (d), (e) and **(f)** show some attractors of the habitat destruction model system with the three species. All graphs in **Figure** (1) are shown the high chaotic behavior of uncontrolled model, which justified our move to study of optimal control for the system.

6.2 The optimal control behavior of the controlled system

In the following part, we will present the optimal trajectory of the optimal controlled mode to any of the equilibrium states of the system Si , $i = 1,2, \dots ,5$ that are defined in (7).

- At $S1 = (0, 0, 0)$

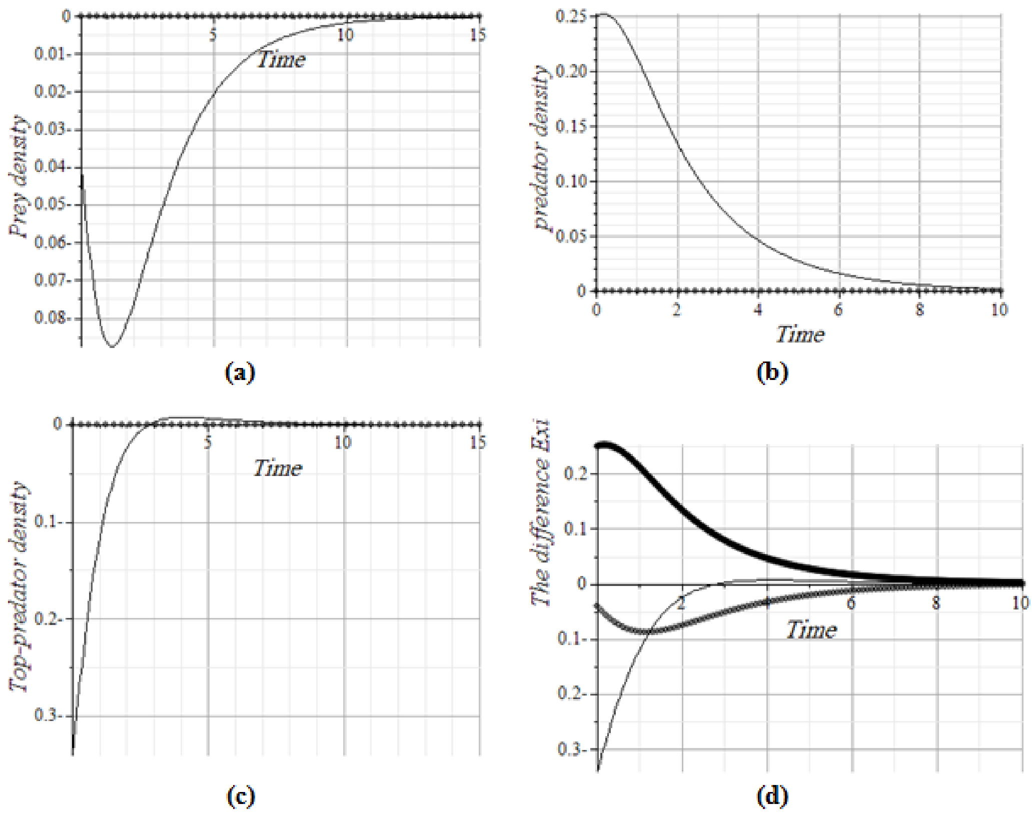
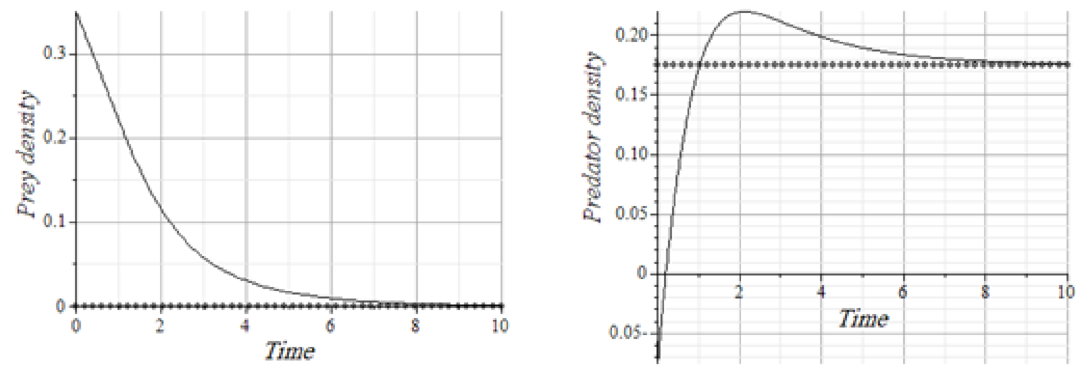


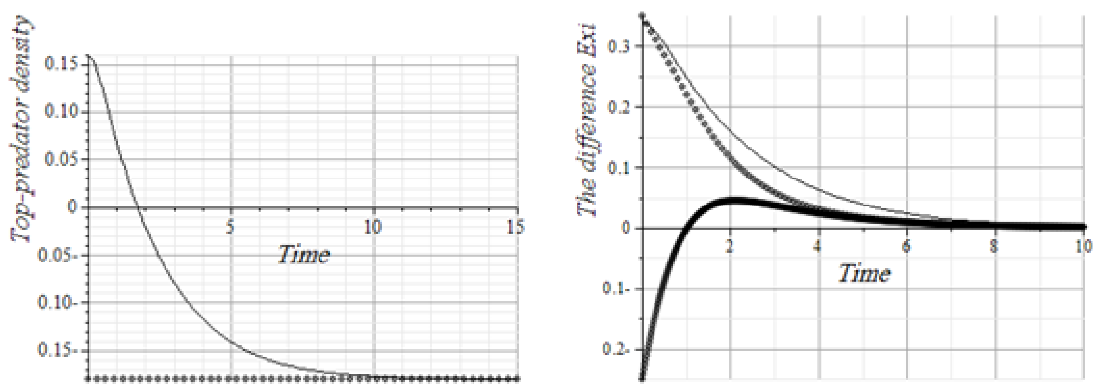
Figure (2): In **(a),(b)** and **(c),** the optimal trajectory behavior of the optimal controlled system to its steady state $S_1 = (0, 0, 0)$, and in **(d)** the simultaneously optimal trajectories of the differences of the vector $(\xi_1, \xi_2, \xi_3)(t)$ at the following probabilities, constants and initial values.

probabilities					Constants		Initial values		
r_0	r_1	r_2	r_3	p	l	k	ξ_{10}	ξ_{20}	ξ_{30}
0.14	0.21	0.13	0.35	0.5	1	4	−0.04	0.25	− 0.34

- At $S_2 = \left(0, \frac{r_3}{2}, -\frac{r_2}{2}\right)$



(a) **(b)**



(c) **(d)**

Figure (3): In **(a),(b)** and **(c),** the optimal trajectory behavior of the optimal controlled system to its steady state $S_2 = (0, 0.175, -0.18)$, and in **(d)** the simultaneously optimal trajectories of the differences vector $(\xi_1, \xi_2, \xi_3)^{(t)}$ at the following probabilities, constants and initial values.

probabilities					Constants		Initial values		
r_0	r_1	r_2	r_3	p	l	k	ξ_{10}	ξ_{20}	ξ_{30}
0.24	0.11	0.35	0.36	0.3	2	8	0.35	− 0.25	0.34

- At $S_3 = \left(\frac{\alpha - r_1}{\alpha},\ 0,\ 0\right)$

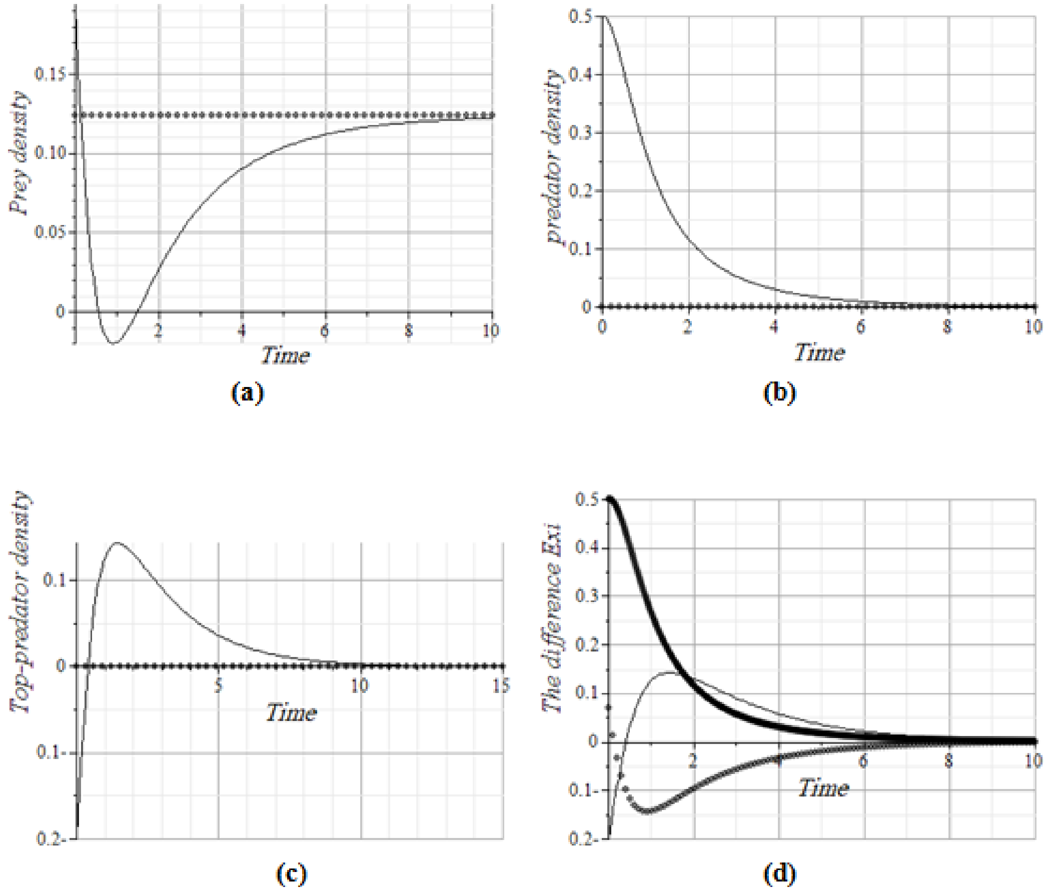


Figure (4): In **(a),(b)** and **(c),** the optimal trajectory behavior of the optimal controlled system to its steady state $S_3 = (0.124,\ 0,\ 0)$, and in **(d)** the simultaneously optimal trajectories of the differences of the vector $(\xi_1, \xi_2, \xi_3)(t)$ at the following probabilities, constants and initial values.

probabilities					Constants		Initial values		
r_0	r_1	r_2	r_3	p	l	k	ξ_{10}	ξ_{20}	ξ_{30}
0.14	0.1	0.13	0.35	0.2	1	4	0.07	0.5	− 0.2

- At $S_4 = \left(\frac{r_2}{2}, \alpha(1 - r_2/2)/(2 + \alpha),\ 0\right)$

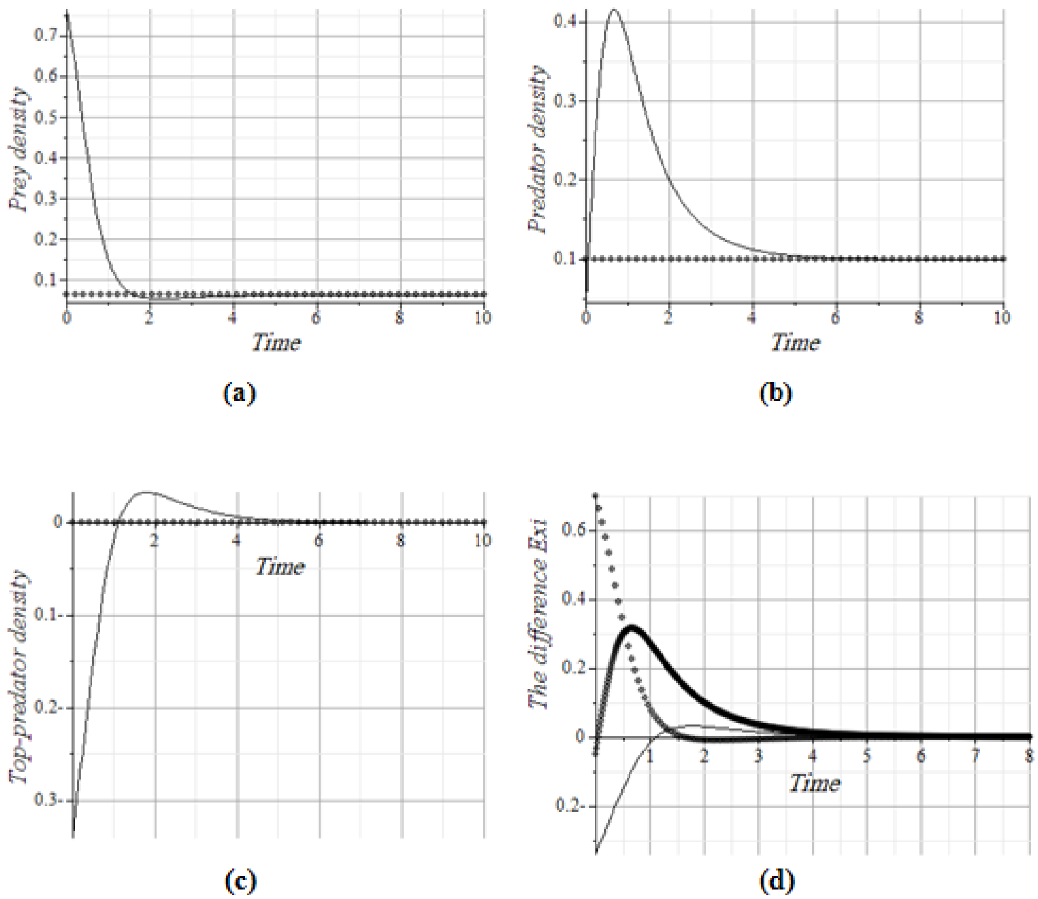


Figure (5): In **(a),(b)** and **(c),** the optimal trajectory behavior of the optimal controlled system to its steady state $S_4 = (0.65, 0.099,\ 0)$, and in **(d)** the simultaneously optimal trajectories of the differences of the vector $(\xi_1, \xi_2, \xi_3)(t)$ at the following probabilities, constants and initial values.

probabilities					Constants		Initial values		
r_0	r_1	r_2	r_3	p	l	k	ξ_{10}	ξ_{20}	ξ_{30}
0.14	0.21	0.13	0.35	0.15	1	1	0.7	−0.5	− 0.34

- At $S_5 = \left(\frac{1}{2} + \frac{r_2 - r_3}{4} - \frac{r_1 + r_3}{2\alpha}, \frac{r_3}{2}, \frac{1}{2} - \frac{r_2 + r_3}{4} - \frac{r_1 + r_3}{2\alpha}\right)$

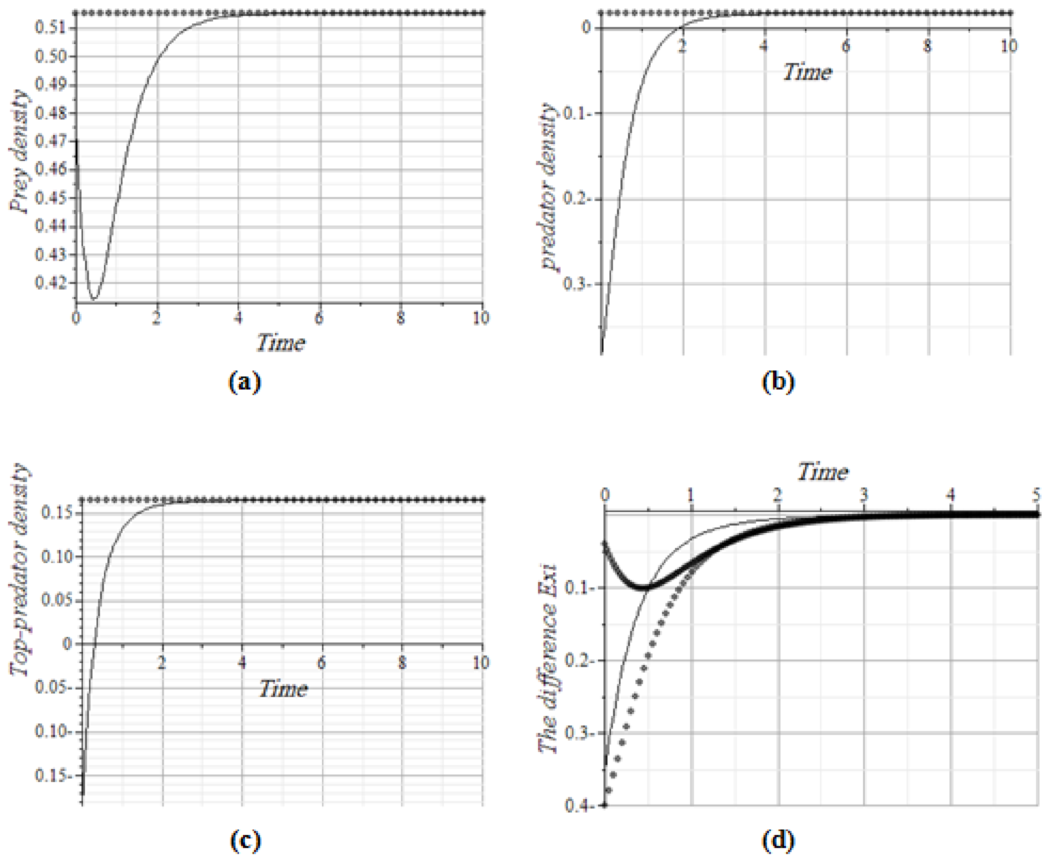


Figure (6): In **(a),(b)** and **(c),** the optimal trajectory behavior of the optimal controlled system to its steady state $S_5 = (0.515, 0.018, 0.165)$, and in **(d)** the simultaneously optimal trajectories of the differences of the vector $(\xi_1, \xi_2, \xi_3)^{(t)}$ at the following probabilities, constants and initial values.

probabilities					Constants		Initial values		
r_0	r_1	r_2	r_3	p	l	k	ξ_{10}	ξ_{20}	ξ_{30}
0.34	0.16	0.7	0.35	0.05	10	4	−0.14	−0.4	− 0.35

In **Figures (2-6) (a, b ,c),** we find that the optimal trajectory densities of prey, predator and top-predator with respect to the objective function in Equation (13) converge over time to their goal levels states (S1 to S5) respectively. The assumed goal levels according the proposed initial and parameters values are represented by the dotted lines. Simultaneously, **Figures (2-6) (d)** show the difference between the densities and the optimal controlled densities of prey, predator and top predator functions with respect to the objective function in Equation (13) and how these deference's converges to zero over time. These results are reflecting the quality and efficiency of the method that was used to create the optimal control inputs. Note that, the top predator density of S2 is absolutely negative, which is biologically inadmissible as we referred above.

6.3 Bifurcation behavior of the system

In this part, we will present the bifurcation behavior of the system at the trivial solution. Choosing $r1$ as a bifurcation parameter, the following figures between $r1$and $S1$ illustrate the bifurcation at the critical point $S1$ = (0, 0, 0).

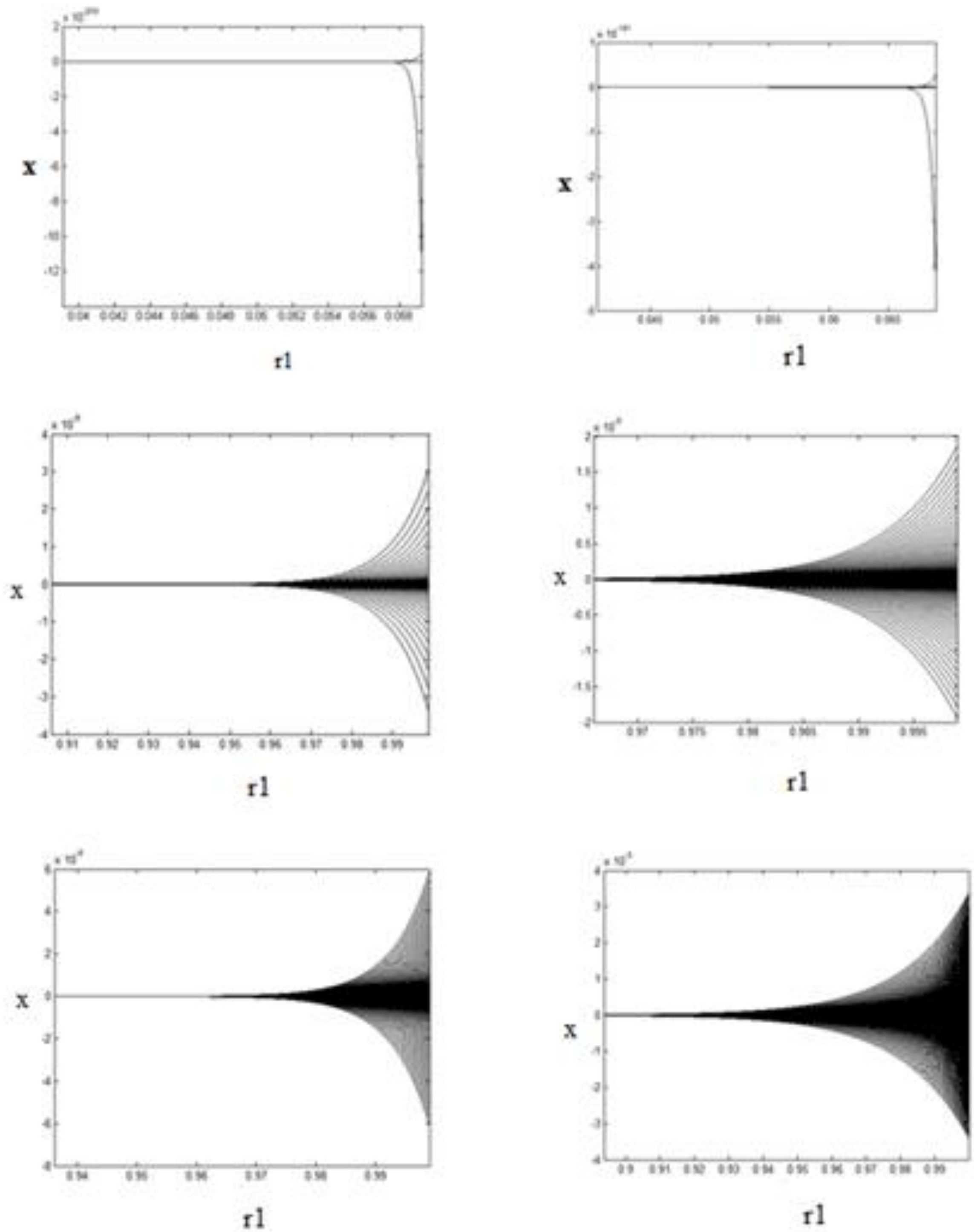


Figure (7**):** The bifurcation diagrams between r_1 and $x$for some minor changes.

7. Concussion

The problem of optimal control problem of habitat destruction model with prey-predator - top predator has been studied. The optimal control functions ensuring asymptotic stability of system steady states are obtained as functions of the phase state and time. The asymptotic stability of these states is proved. The bifurcation phenomena at the trivial solution of the uncontrolled system has discussed and presented graphically. Analysis and simulation study and numerical examples are presented.

References

[1]Szwabinski J. and Pekalski A. (2006), Effects of random habitat destruction in a predator-prey model, *Physica A, 360, 59-70*.

[2]Nakagiri, N., Tainaka, K. *(2004), Indirect effects of habitat destruction in model cosystems, Ecological Modelling 174, 103-114*.

[3]Szwabinski, J. and Pekalski, A. *(2006), Effects of random habitat destruction in a predator-prey model, Physica A 360, 59-70*.

[4]Nakagiri, N., Tainaka, K., Tao, T. *(2001), Indirect relation between extinction and habitat destruction. Ecol. Mod. 137 (a) 109-118*.

[5]El-Gohary A. (2005), Optimal control of stochastic lattice of prey-predator models, *Applied Mathematics and Computation, 160, 15-28*.0

[6]Klausmeier C.A. (1998), Extinction in Multispecies and Spatially Explicit Models of Habitat Destruction, *The american naturalist, 152(2).*

[7]Swihart R., Feng Z., Slade N., Mason D. and Gehring T. (2001), Effects of Habitat Destruction and Resource Supplementation in a Predator-Prey Metapopulation Model, *Journal of Theoretical Biology, 210, 287-303.*

[8]*Saba Alwan* and Omalsad Hamood Odhah. Stability, Chaos and Estimation of the Unknown Parameters of Habitat Destruction Model with Prey-Predator - Top Predator. Journal of Applied Mathematics and Physics. (2016) 4, 2254 - 2271.

[9]Alwan. Saba and El-Gohary A. (2011), *Chaos, estimation and optimal control of habitat destruction model with uncertain parameters*, Computers and Mathematics with Applications 62 , 4089–4099.

[10] Alwan S., (2016) , Chaos Behavior and Estimation of the Unknown Parameters of Stochastic Lattice Gas for Prey-Predator Model Pair-Approximation, Applied Mathematics, 2016, 7, 1765-1779.

[11] Al-Mahdi A. and Khirallah M., (2016) Bifurcation Analysis of a Model of Cancer, European Scientific Journal, Vol.12 No 3 , 68-83 .

[12] Khirallah M. and Al-Mahdi A., (2014) Stability Analysis of A model of Cancer Treatment by Immunotherapy, European Journal of Scientific Research, Vol.121 No 2 , 161-173.

[13] N. Krasovskii, Problems in the stabilization of controlled motion, in: I.G. Malkin (Ed.), The Stability of Motion, Nauka Press, Moscow, 1966.