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Numerical study of three dimensional partial differential equations with variable coefficients by RDTM and comparison with DTM

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Abstract
In this paper, we apply Reduced Differential Transform Method (RDTM) and compared with Differential Transform Method (DTM) for solving partial differential equations in three dimensions with variable coefficients, numerical applications are shown for implementing those methods. The results show that RDTM is very effective and simple.

Keywords: Differential transform method, reduced differential transform method, heat-like and wave-like equations, and analytic solution

Introduction
Many physical problems can be described by mathematical models that involve partial differential equations. A mathematical model is a simplified description of physical reality expressed in mathematical terms. Thus, the investigation of the exact or approximation solution helps us to understand the means of these mathematical models. Several numerical methods were developed for solving partial differential equations with variable coefficients such us He's Polynomials [1], the homotopy perturbation method [2], homotopy analysis method [3], the modified variational iteration method [4],differential transform method [11] and reduced differential transform method[12].
The main goal of this paper is to apply reduced differential transform method (RDTM) [5-9], [12] and compared with the differential transform method (DTM)[11], to obtain the exact solution for heat and wave-like equations[10]: the heat-like equation in three dimensions of the form

$$u_t + f_1(x,y,z)u_{xx} + f_2(x,y,z)u_{yy} + f_3(x,y,z)u_{zz} = 0$$

with the initial condition

$$u(x,y,z,0) = f_4(x,y,z),$$

and the wave-like equation in three dimension of the form

$$u_{tt} + g_1(x,y,z)u_{xx} + g_2(x,y,z)u_{yy} + g_3(x,y,z)u_{zz} = 0$$

with the initial condition

$$u(x,y,z,0) = g_4(x,y,z) \ , \ u_t(x,y,z,0) = g_5(x,y,z)$$

The proving that the RDTM is very efficient, suitable, quite accurate and simple to such type of equations.

Analysis of the methods

To illustrate the basic idea of the DTM, we considered $u(x,t)$ is analytic and differentiated continuously in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0}, \quad \text{…. (1)}$$

where the spectrum U_k (x) is the transformed function, which is called T-function in brief. The differential inverse transform of U_k (x) is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)(t-t_0)^k , \quad \text{…... (2)}$$

Combining (1) and (2), it can be obtained that

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0} (t-t_0)^k , \quad \text{……. (3)}$$

when (t_0)are taken as ($t_0 = 0$)then Equation (3) is expressed as

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} t^k , \quad \text{…… (4)}$$

and Equation (2) is shown as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k , \quad \text{…… (5)}$$

In real application, the function $u(x,t)$ by a finite series of Equation (5) can be written as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k , \quad \text{……. (6)}$$

usually, the values of n is decided by convergence of the series coefficients.

The fundamental mathematical operations performed by n-Dimensional Differential Transform are listed in Table 1.

Table 1. The fundamental operations of n-dimensional DTM.

Original function	Transformed function
$w(x_1,...,x_n) = \alpha\, u(x_1,...,x_n) \pm \beta\, v(x_1,...,x_n)$	$W(k_1,...,k_n) = \alpha\, U(k_1,...,k_n) \pm \beta\, V(k_1,...,k_n)$
$w(x_1,x_2,...,x_n) = \frac{\partial^{r_1+r_2+\dots+r_n} u(x_1,x_2,...,x_n)}{\partial x_1^{r_1} \dots \partial x_n^{r_n}}$	$W(k_1,...,k_n) = (k_1+1)...(k_1+r_1)(k_2+1)...$ $(k_2+r_2)...(k_n+1)...(k_n+r_n)$ $U(k_1+r_1,k_2+r_2...,k_n+r_n)$
$w(x_1,x_2,...,x_n) = x_1^{e_1} x_2^{e_2} ... x_n^{e_n}$	$W(k_1,...,k_n) = \delta(k_1-e_1,k_2-e_2,...,k_n-e_n)$

The basic definitions of reduced differential transform method are introduced as follows:
Definition 1 [5-9]

If the function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0}, \qquad (1)$$

where the t -dimensional spectrum function $U_k(x)$ is the transformed function. In this paper, the lowercase represent the original function $u(x,t)$ while the uppercase $U_k(x)$ stand for the transformed function

Definition 2 [5-9]

The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k, \qquad (2)$$

Then combining equation (1) and (2) we write

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} t^k, \qquad (3)$$

From the above definitions, it can be found that the concept of the reduced differential transform is derived from the power series expansion. The fundamental mathematical operations performed by RDTM can be readily obtained and are listed in Table 2.

Table 2: Reduced differential transform

Functional Form	Transformed Form
$u(x,t)$	$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}$
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = \alpha\, u(x,t)$	$W_k(x) = \alpha\, U_k(x)$ α *is a constant*
$w(x,t) = x^m t^n$	$W_k(x) = x^m \delta(k-n),\ \delta(k) = \begin{cases}1 & ,k=0\\ 0 & ,k\neq 0\end{cases}$
$w(x,t) = x^m t^n u(x,t)$	$W_k(x) = x^m U_{k-n}(x)$
$w(x,t) = u(x,t)\, v(x,t)$	$W_k(x) = \sum_{r=0}^{k} V_r U_{k-r}(x) = \sum_{r=0}^{k} U_r V_{k-r}(x)$
$w(x,t) = \frac{\partial^r}{\partial t^r}u(x,t)$	$W_k(x) = (k+1)\ldots(k+r)U_{k+r}(x)$
$w(x,t) = \frac{\partial}{\partial x}u(x,t)$	$W_k(x) = \frac{\partial}{\partial x}\, U_k(x)$
$w(x,t) = \frac{\partial^2}{\partial x^2}u(x,t)$	$W_k(x) = \frac{\partial^2}{\partial x^2}\, U_k(x)$

Numerical applications

In this section, we apply differential transform method (DTM) and reduced differential transform method (RDTM) for solving heat and wave-like equations

Example 1: We considerthe three-dimensional heat-like model

$$u_t = x^4y^4z^4 + \frac{1}{36}(x^2u_{xx} + y^2u_{yy} + z^2u_{zz}), \quad 0 < x, y, z < 1, \; t > 0, \qquad \text{.... (1)}$$

subject to the boundary conditions

$$\begin{aligned} u(0,y,z,t) &= 0, & u(1,y,z,t) &= y^4z^4(e^t-1), \\ u(x,0,z,t) &= 0, & u(x,1,z,t) &= x^4z^4(e^t-1), \\ u(x,y,0,t) &= 0, & u(x,y,1,t) &= x^4y^4(e^t-1), \end{aligned} \qquad \text{..... (2)}$$

and the initial condition $u(x,y,z,0) = 0\,.$ ..... (3)

Taking the four dimensional differential transform of (1), we can obtain:

$$\begin{aligned} U(k,h,n,m+1) &= \tfrac{1}{m+1}\delta(k-4,h-4,n-4,m) \\ &+ \tfrac{1}{36(m+1)}\Bigg[\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-2,s,\lambda,m-l)(k-r+1)(k-r+2)U(k-r+2,h-s,n-\lambda,l) \\ &+ \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s-2,\lambda,m-l)(h-s+1)(h-s+2)U(k-r,h-s+2,n-\lambda,l) \\ &+ \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s,\lambda-2,m-l)(n-\lambda+1)(n-\lambda+2)U(k-r,h-s,n-\lambda+2,l)\Bigg]. \end{aligned} \qquad \text{... (4)}$$

By applying the differential transform to boundary conditions (2), we have

$$U(0,h,n,m) = 0\,,$$

$$U(1,h,n,m) = \left(\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s-4,\lambda-4,m-l).\tfrac{1}{(k-r+h-s+n-\lambda+l)!}\right) - \delta(r,s-4,\lambda-4,m-l)\,,$$

$$U(k,0,n,m) = 0\,,$$

$$U(k,1,n,m) = \left(\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-4,s,\lambda-4,m-l).\tfrac{1}{(k-r+h-s+n-\lambda+l)!}\right) - \delta(r-4,s,\lambda-4,m-l)\,,$$

$$U(k,h,0,m) = 0\,,$$

$$U(k,h,1,m) = \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-4,s-4,\lambda,m-l).\tfrac{1}{(k-r+h-s+n-\lambda+l)!} - \delta(r-4,s-4,\lambda,m-l)\,. \qquad \text{... (5)}$$

From the initial condition (3), we can write

$$U(k,h,n,0) = 0\,. \qquad \text{... (6)}$$

For each k, h substituting equations (5) and (6) into equation (4) and by recursive method, the values U (k, h) can be evaluated as follows:

$$U(k,h,n,m+1)=0\,, \quad for\ \ k=0,1,2,3,5,6,\dots\ ,\ h=n=m=0,1,2,... \qquad \dots (7)$$

$$U(4,4,4,m)=\begin{cases} 0 & m=0 \\ \frac{1}{m!} & m\geq 1. \end{cases} \qquad ... (8)$$

By using the inverse transformation rule for four dimensional in table 1, we obtain series for $u(x, y, z, t)$. When we rearrange the solution, we get the following closed form solution:

$$\begin{aligned} u(x,y,z,t) &= x^4y^4z^4(t+\tfrac{t^2}{2!}+\tfrac{t^3}{3!}+\tfrac{t^4}{4!}+...) \\ &= x^4y^4z^4(e^t-1)\,. \end{aligned} \qquad \dots (9)$$

Similarly, by using the reduced differential transform to (1), we obtain the recurrence equation

$$(k+1)\ U_{k+1}(x,y,z) = (xyz)^4 - \frac{1}{36}\Big(x^2\frac{\partial^2}{\partial x^2}U_k(x,y,z) + y^2\frac{\partial^2}{\partial y^2}U_k(x,y,z) + z^2\frac{\partial^2}{\partial z^2}U_k(x,y,z) = 0 \qquad ... (10)$$

From the initial condition by given of (3)

$$U_0(x,y,z)=0 \qquad ... (11)$$

Substituting (10) and (11) , we have

$$U_1(x,y,z)= x^4\ y^4\ z^4\,, U_2(x,y,z)=\frac{1}{2}x^4\ y^4\ z^4\,, U_3(x,y,z)=\frac{1}{6}x^4\ y^4\ z^4,$$

$$U_4(x,y,z)=\frac{1}{24}x^4\ y^4\ z^4, U_5(x,y,z)=\frac{1}{120}x^4\ y^4\ z^4, U_6(x,y,z)=\frac{1}{720}x^4\ y^4\ z^4$$

$$, \dots, U_k(x,y,z)=\frac{1}{k!}x^4\ y^4\ z^4.$$

The series solution is given by

$$u(x,y,z,t)=x^4\ y^4\ z^4(t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+.....+\frac{t^k}{k!}),$$

$$u(x,y,z,t)=x^4\ y^4\ z^4(e^t-1).$$

This is the exact solution.

Example 2: We consider the three-dimensional wave-like model

$$u_{tt}=\left(x^2+y^2+z^2\right)+\frac{1}{2}(x^2u_{xx}+y^2u_{yy}+z^2u_{zz})\,, \quad 0<x,y,z<1,\ t>0\,, \qquad \dots (1)$$

subject to the boundary conditions

$$u(0,y,z,t) = y^2(e^t-1)+z^2(e^{-t}-1), \quad u(1,y,z,t) = \left(1+y^2\right)\left(e^t-1\right)+z^2(e^{-t}-1),$$
$$u(x,0,z,t) = x^2(e^t-1)+z^2(e^{-t}-1), \quad u(x,1,z,t) = \left(1+x^2\right)\left(e^t-1\right)+z^2(e^{-t}-1),$$
$$u(x,y,0,t) = \left(x^2+y^2\right)\left(e^t-1\right), \qquad u(x,y,1,t) = \left(x^2+y^2\right)\left(e^t-1\right)+\left(e^{-t}-1\right), \qquad \text{... (2)}$$

and the initial conditions

$$u(x,y,z,0) = 0\,, \qquad u_t(x,y,z,0) = x^2+y^2-z^2. \qquad \text{… (3)}$$

Taking the four dimensional differential transform of (1), we can obtain:

$$U(k,h,n,m+2) = \tfrac{1}{(m+1)(m+2)}\left[\delta(k-2,h,n,m)+\delta(k,h-2,n,m)+\delta(k,h,n-2,m)\right]$$

$$+\tfrac{1}{2(m+1)(m+2)}\left[\begin{array}{l} \sum\limits_{r=0}^{k}\sum\limits_{s=0}^{h}\sum\limits_{\lambda=0}^{n}\sum\limits_{l=0}^{m}\delta(r-2,s,\lambda,m-l)(k-r+1)(k-r+2)U(k-r+2,h-s,n-\lambda,l) \\ +\sum\limits_{r=0}^{k}\sum\limits_{s=0}^{h}\sum\limits_{\lambda=0}^{n}\sum\limits_{l=0}^{m}\delta(r,s-2,\lambda,m-l)(h-s+1)(h-s+2)U(k-r,h-s+2,n-\lambda,l) \\ +\sum\limits_{r=0}^{k}\sum\limits_{s=0}^{h}\sum\limits_{\lambda=0}^{n}\sum\limits_{l=0}^{m}\delta(r,s,\lambda-2,m-l)(n-\lambda+1)(n-\lambda+2)U(k-r,h-s,n-\lambda+2,l) \end{array}\right] \qquad \text{…. (4)}$$

By applying the differential transform to boundary conditions (2), we get

$$\begin{aligned} U(0,h,n,m) &= \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s-2,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h-2,n,m) \\ &\quad+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s,\lambda-2,m-l)\cdot\tfrac{(-1)^{k-r+h-s+n-\lambda+l}}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h,n-2,m), \end{aligned}$$

$$\begin{aligned} U(1,h,n,m) &= \tfrac{1}{m!}-\delta(k,h,n,m)+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s-2,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!} \\ &\quad-\delta(k,h-2,n,m)+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s,\lambda-2,m-l)\cdot\tfrac{(-1)^{k-r+h-s+n-\lambda+l}}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h,n-2,m), \end{aligned}$$

$$\begin{aligned} U(k,0,n,m) &= \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-2,s,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!}-\delta(k-2,h,n,m) \\ &\quad+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s,\lambda-2,m-l)\cdot\tfrac{(-1)^{k-r+h-s+n-\lambda+l}}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h,n-2,m), \end{aligned}$$

$$\begin{aligned} U(k,0,n,m) &= \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-2,s,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!}-\delta(k-2,h,n,m) \\ &\quad+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s,\lambda-2,m-l)\cdot\tfrac{(-1)^{k-r+h-s+n-\lambda+l}}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h,n-2,m), \end{aligned}$$

$$\begin{aligned} U(k,1,n,m) &= \tfrac{1}{m!}-\delta(k,h,n,m)+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-2,s,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!} \\ &\quad+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s,\lambda-2,m-l)\cdot\tfrac{(-1)^{k-r+h-s+n-\lambda+l}}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h,n-2,m), \end{aligned}$$

$$\begin{aligned} U(k,h,0,m) &= \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-2,s,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!}-\delta(k-2,h,n,m) \\ &+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s-2,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h-2,n,m), \\ U(k,h,1,m) &= \sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r-2,s,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!}-\delta(k-2,h,n,m) \\ &+\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{\lambda=0}^{n}\sum_{l=0}^{m}\delta(r,s-2,\lambda,m-l)\cdot\tfrac{1}{(k-r+h-s+n-\lambda+l)!}-\delta(k,h-2,n,m) \\ &+\tfrac{(-1)^m}{m!}-\delta(k,h,n,m). \end{aligned}$$

… (5)

From the initial condition (3), we can write

$$U(k,h,n,0)=0,$$

$$U(k,h,n,1)=\delta(k-2,h,n,m)+\delta(k,h-2,n,m)-\delta(k,h,n-2,m)$$

$$= \begin{cases} 1 & k=2, h=n=m=0 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} 1 & h=2,\ \ k=n=m=0 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} 1 & n=2,\ \ k=h=m=0 \\ 0 & \text{otherwise.} \end{cases}$$

…. (6)

For each *k, h*, substituting equations (5) and (6) into equation (4), and via the recursive method, the values *U* (*k, h*) can be evaluated as follows:

$$U(k,h,n,m+1)=0\,, \quad for\ \ k=1,3,4,5,6,\dots\ ,\ \ h=n=m=0,1,2,\dots$$

$$U(0,0,2,m)=\begin{cases} 0 & m=0 \\ \frac{(-1)^m}{m!} & m\geq 1 \end{cases}$$

$$U(0,2,0,m)=\begin{cases} 0 & m=0 \\ \frac{1}{m!} & m\geq 1 \end{cases}$$

$$U(2,0,0,m)=\begin{cases} 0 & m=0 \\ \frac{1}{m!} & m\geq 1. \end{cases}$$

…. (7)

Using the inverse transformation rule for four dimensional in table 1, we obtain a series for *u*(*x, y, z, t*).

On rearranging the solution, we get the following closed form solution:

$$\begin{aligned} u(x,y,z,t) &= z^2\left(-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}\cdots\right)+y^2\left(t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}\cdots\right)+x^2\left(t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}\cdots\right) \\ &= z^2\left(e^{-t}-1\right)+\left(y^2+x^2\right)\left(e^t-1\right). \end{aligned}$$

That is,

$$u(x,y,z,t) = z^2\sum_{j=1}\frac{(-t)^j}{j!}+y^2\sum_{j=1}\frac{t^j}{j!}+x^2\sum_{j=1}\frac{t^j}{j!} = z^2(e^{-t}-1)+(y^2+x^2)(e^t-1).$$

Similarly , By using the reduced differential transform to (1), we obtain the recurrence equation

$$(k+1)(k+2)U_{k+2}(x,y,z)-\frac{x^2}{12}\frac{\partial^2}{\partial x^2}U_k(x,y,z)-\frac{y^2}{12}\frac{\partial^2}{\partial y^2}U_k(x,y,z)=0 \quad , \qquad \ldots. (8)$$

From the initial condition (3), we write

$$U_0(x,y,z)=0, \quad U_1(x,y,z)=x^2+y^2-z^2, \qquad \ldots. (9)$$

Substituting (9) into (8), we have

$$U_2(x,y,z)=\frac{1}{2}\,(x^2+y^2+z^2),\, U_3(x,y,z)=\frac{1}{6}\,(x^2+y^2-z^2)\,,\, U_4(x,y,z)=\frac{1}{24}\,(x^2+y^2+z^2),$$

$$U_5(x,y,z)=\frac{1}{120}\,(x^2+y^2-z^2),U_6(x,y,z)=\frac{1}{720}(x^2+y^2-z^2),\cdots.....$$

From definition 2,we obtained the closed from series solution as

$$u(x,y,z,t)=\sum_{k=0}^{\infty}U_k(x,y,z)\,t^k$$

$$=(x^2+y^2+z^2)\sum_{k=0,2,4,}^{\infty}U_k(x,y,z)\,t^k+(x^2+y^2-z^2)\sum_{k=1,3,5,}^{\infty}U_k(x,y,z)\,t^k$$

$$u(x,y,t)=(x^2+y^2)(t+\frac{t^2}{2!}+\frac{t^3}{3!}+\cdots)+z^2(-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\cdots)$$

Then, the exact solution is given in the closed form by

$$u(x,y,z,t)=(x^2+y^2)\,e^t+z^2e^{-t}-(x^2+y^2+z^2)$$

Conclusion

In this research, reduced differential transform method has been applied and compared with DTM in solving the three dimensional heat and wave-like equations. RDTM is applied in a direct way without using linearization, transformation, discretization or restrictive assumptions. The results shows that RDTM is powerful and efficient techniques in finding the exact solution and has small size of computation contrary to differential transform method (DTM),

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