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Synchronization of a Planner Magnetic Binaries Problem Including the effect of the Gravitational Forces of the Primaries

Mohd. Arif*, Ravi Kumar Garg

Department of Mathematics, Zakir Husain Delhi College, Delhi University, New Delhi, India.

Correspondence Address: *Mohd. Arif, Department of Mathematics, Zakir Husain Delhi College, Delhi University, New Delhi, India.

Abstract

In this article we have investigated the synchronization behavior of the planar magnetic-binaries problem including the effect of the gravitational forces of the primaries on the small body evolving from deferent initial conditions using active control technique based on the Lyapunov-stability theory and Routh-Hurwitz criteria. Numerical simulations are performed to plot phase portraits, time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control techniques.

Keywords: Space dynamics, magnetic-binaries problem, Synchronization, Lyapunov stability theory and Routh- Hurwitz criteria

Introduction

In the last two decades considerable research has been done in non-linear dynamical systems and their various properties. One of the most important properties is synchronization which is an important topic in the nonlinear dynamics. Generally speaking, the synchronization phenomenon has the following feature: the trajectories of two systems (master and slave systems) are identical notwithstanding starting from different initial conditions. Chaotic synchronization did not attract much attention until Pec-ora and Carroll introduced a method to synchronize two identical chaotic systems with deferent initial conditions in 1990 and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Chaos synchronization using active control was proposed by Bai and Lonngren in 1997 and has recently been widely accepted as an efficient technique for the synchronization of chaotic systems, because it can be used to synchronize non-identical systems as well; a feature that gives it an advantage over other synchronization methods. Many methods have been developed to synchronize chaotic systems, including nonlinear feedback method (L. Lu, C. Zhang and Z.A. Guo 2007), adaptive control method (Y.Wang, Z.H. Guan and H.O. Wang 2003), anti-synchronization method (J.H. Park, 2006) and sliding mode control method (M. Haeri and A. Emadzadeh 2007). Chaos synchronization using active control which is introduced in 1998 is one of the these methods. Unified chaotic systems, the energy resource chaotic system and some other systems have been synchronized with this method. In 2013 Ayub Khan and Priyamvada Tripathi have investigated the synchronization behavior of a restricted three body problem under the effect of radiation pressure.

Stormer (1907) has studied the motion of a charged particle which is moving in the field of a magnetic diploe as a two body problem. This problem in general is quite complicated and is a non integrable. A. Mavragnais (1978, 1988) has studied the motion of a charge particle which is moving in the field of two rotating magnetic dipoles instead of one dipole.

In this article, active control techniques base on the Lyapunov stability theory and Routh-Hurwitz criteria have been used to study the synchronization behavior of planar magnetic- binaries problem including the effect of the gravitational forces of the primaries on the small body in a particular case. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with defferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

The aim of this study to investigate the synchronization behavior of the planar magnetic- binaries problem including the effect of the gravitational forces of the primaries on the small body and transferring the origin of the coordinate system to the second mass .

Equation of motion

Two bodies (the primaries), with magnetic fields move under the influence of gravitational force and a charged particle P (small body) of charge q and mass m moves in the vicinity of these bodies. The question of the magnetic-binaries problem is to describe the motion of this particle. The equation of motion and the integral of relative energy in the rotating coordinate system including the effect of the gravitational forces of the primaries on the small body written as:

$$\ddot{x} - \dot{y}\,f = U_x \qquad (1)$$

$$\ddot{y} + \dot{x}\,f = U_y \qquad (2)$$

$$\dot{x}^2 + \dot{y}^2 = 2U - C \qquad (3)$$

Where

$$f = 2 - \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right), \; U_x = \frac{\partial U}{\partial x} \text{ and } U_y = \frac{\partial U}{\partial y}$$

$$U = (x^2 + y^2)\left\{\frac{1}{2} + \frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} + x\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \qquad (4)$$

Here we assumed

1. Primaries participate in the circular motion around their centre of mass

2. Position vector of P at any time t be $\overline{r} = (xi + yj + zk)$ referred to a rotating frame of reference $O(x, y, z)$ which is rotating with the same angular velocity $\overline{\omega} = (0, 0, 1)$ as those the primaries.

3. Initially the primaries lie on the x-axis.

4. The distance between the primaries as the unit of distance and the coordinate of one primary is $(\mu, 0, 0)$ then the other is $(\mu - 1, 0, 0)$.

5. The sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is $(1 - \mu)$.

6. The unit of time in such a way that the gravitational constant G has the value unity and q= mc where c is the velocity of light.

$r_1^2 = (x-\mu)^2 + y^2$, $r_2^2 = (x+1-\mu)^2 + y^2$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).

Therefore, instead of dealing with the full equations of planar magnetic-binaries problem, it makes more sense to work with a system of equations that describe the motion of the small body in the vicinity of the secondary mass, this type of system was derived by Hill in 1878. By making some assumptions and transferring the origin of the coordinate system to the second mass fig(1) the equations of motion (1) and (2), become

$$\ddot{\zeta} - \dot{\nu} f = U_\zeta \tag{5}$$

$$\ddot{\nu} + \dot{\zeta} f = U_\nu \tag{6}$$

Where

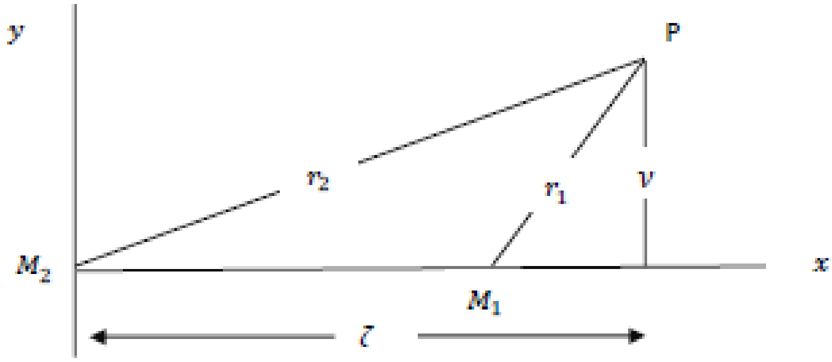
$$f = 2 - \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right),\ U_\zeta = \frac{\partial U}{\partial \zeta} \text{ and } U_\nu = \frac{\partial U}{\partial \nu}$$

$$U = \{(\zeta + \mu - 1)^2 + \nu^2\}\left\{\frac{1}{2} + \frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} - (\zeta + \mu - 1)\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \tag{7}$$

$$r_1^2 = (\zeta - 1)^2 + \nu^2,\quad r_2^2 = \zeta^2 + \nu^2$$

And new Jacobi constant, C_n, is given by

$$\dot{\zeta}^2 + \dot{\nu}^2 = 2U - C_n \tag{8}$$



Fig(1)

3 Synchronization via Active Control

Let

$\zeta = x_1,\ \dot{\zeta} = x_2,\ \nu = x_3,\ \dot{\nu} = x_4$

Then the equation (5) and (6) can be written as:

$$
\left.\begin{aligned}
&\dot{x_1} = x_2\\
&\dot{x_2} = x_4\left\{2-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}+(x_1+\mu-1)\left[1-\left\{\frac{3(x_1-1)^2}{r_1^5}+\frac{3\lambda x_1^2}{r_2^5}-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}\right]-\\
&-3x_3^2\left\{\frac{(x_1-1)}{r_1^5}++\frac{\lambda x_1}{r_2^5}\right\}+\frac{(x_1-1)}{r_1^3}+\frac{\lambda x_1}{r_2^3}-\frac{(1-\mu)(x_1-1)}{r_1^3}-\frac{\mu x_1}{r_2^3}\\
&\dot{x_3} = x_4\\
&\dot{x_4} = -x_2\left\{2-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}+x_3-3x_3(x_1+\mu-1)\left\{\frac{(x_1-1)}{r_1^5}+\frac{\lambda x_1}{r_2^5}\right\}-\\
&-3x_3^3\left\{\frac{1}{r_1^5}++\frac{\lambda}{r_2^5}\right\}+2x_3\left\{\frac{1}{r_1^3}+\frac{\lambda}{r_2^3}\right\}-\frac{x_3(1-\mu)}{r_1^3}-\frac{\mu x_3}{r_2^3}
\end{aligned}\right\} \quad (9)
$$

Where

$r_1^2 = (x_1-1)^2+{x_3}^2,\ \ r_2^2 = {x_1}^2+{x_3}^2$

Corresponding to master system (9), the identical slave system is defined as:

$$\left.\begin{aligned}
\dot{y_1} &= y_2 + u_1(t)\\
\dot{y_2} &= y_4\left\{2-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}+(y_1+\mu-1)\left[1-\left\{\frac{3(y_1-1)^2}{r_1^5}+\frac{3\lambda y_1^2}{r_2^5}-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}\right]-\\
&-3y_3^2\left\{\frac{(y_1-1)}{r_1^5}+\ +\frac{\lambda y_1}{r_2^5}\right\}+\frac{(y_1-1)}{r_1^3}+\frac{\lambda y_1}{r_2^3}-\frac{(1-\mu)(y_1-1)}{r_1^3}-\frac{\mu y_1}{r_2^3}+u_2(t)\\
\dot{y_3} &= y_4 + u_3(t)\\
\dot{y_4} &= -y_2\left\{2-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}+y_3-3y_3(y_1+\mu-1)\left\{\frac{(y_1-1)}{r_1^5}+\frac{\lambda y_1}{r_2^5}\right\}-\\
&3y_3^3\left\{\frac{1}{r_1^5}+\ +\frac{\lambda}{r_2^5}\right\}+2y_3\left\{\frac{1}{r_1^3}+\frac{\lambda}{r_2^3}\right\}-\frac{y_3(1-\mu)}{r_1^3}-\frac{\mu y_3}{r_2^3}+u_4(t)
\end{aligned}\right\}\qquad(10)$$

Where

$r_1^2 = (y_1-1)^2 + y_3^2, \quad r_2^2 = y_1^2 + y_3^2$

where $u_i(t)$; i =1 ,2,3,4 are control functions to be determined.

Let $e_i = y_i - x_i$; i = 1, 2, 3, 4 be the synchronization errors (Liu 2004). From (9) and (10), we obtain the error dynamics as follows:

$$\left.\begin{aligned}
\dot{e_1} &= e_2 + u_1(t)\\
\dot{e_2} &= 2e_4 + e_1 - y_4\left\{\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}-(y_1+\mu-1)\left\{\frac{3(y_1-1)^2}{r_1^5}+\frac{3\lambda y_1^2}{r_2^5}-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}-\\
&\qquad -3y_3^2\left\{\frac{(y_1-1)}{r_1^5}+\ +\frac{\lambda y_1}{r_2^5}\right\}-3y_3^2\left\{\frac{(y_1-1)}{r_1^5}+\ +\frac{\lambda y_1}{r_2^5}\right\}+\frac{(y_1-1)}{r_1^3}+\frac{\lambda y_1}{r_2^3}-\\
&-\frac{(1-\mu)(y_1-1)}{r_1^3}-\frac{\mu y_1}{r_2^3}+x_4\left\{\frac{1}{r_1^3}+\frac{\lambda}{r_2^3}\right\}+(x_1+\mu-1)\left\{\frac{3(x_1-1)^2}{r_1^5}+\frac{3\lambda x_1^2}{r_2^5}-\frac{1}{r_1^3}-\frac{\lambda}{r_2^3}\right\}+\\
&3x_3^2\left\{\frac{(x_1-1)}{r_1^5}+\ +\frac{\lambda x_1}{r_2^5}\right\}-\frac{(x_1-1)}{r_1^3}-\frac{\lambda x_1}{r_2^3}+\frac{(1-\mu)(x_1-1)}{r_1^3}+\frac{\mu x_1}{r_2^3}+u_2(t)\\
\dot{e_3} &= e_4 + u_3(t)\\
\dot{e_4} &= -2e_2 + e_3 + y_2\left\{\frac{1}{r_1^3}+\frac{\lambda}{r_2^3}\right\}-3y_3(y_1+\mu-1)\left\{\frac{(y_1-1)}{r_1^5}+\frac{\lambda y_1}{r_2^5}\right\}-3y_3^3\left\{\frac{1}{r_1^5}+\ +\frac{\lambda}{r_2^5}\right\}+\\
&+2y_3\left\{\frac{1}{r_1^3}+\frac{\lambda}{r_2^3}\right\}-\frac{y_3(1-\mu)}{r_1^3}-\frac{\mu y_3}{r_2^3}+x_2\left\{\frac{1}{r_1^3}+\frac{\lambda}{r_2^3}\right\}+3x_3(x_1+\mu-1)\left\{\frac{(x_1-1)}{r_1^5}+\frac{\lambda x_1}{r_2^5}\right\}+\\
&3x_3^3\left\{\frac{1}{r_1^5}+\ +\frac{\lambda}{r_2^5}\right\}-2x_3\left\{\frac{1}{r_1^3}+\frac{\lambda}{r_2^3}\right\}+\frac{x_3(1-\mu)}{r_1^3}+\frac{\mu x_3}{r_2^3}+u_4(t)
\end{aligned}\right\}\qquad(11)$$

This above error system to be controlled is a linear system with control functions. Thus, let us redefine the control functions so that the terms in (11) which cannot be expressed as linear terms in ei 's are eliminated:

$$\left.\begin{aligned}
&u_1(t) = v_1(t)\\
&u_2(t) = y_4\left\{\frac{1}{r_1^3} - \frac{\lambda}{r_2^3}\right\} + (y_1+\mu-1)\left\{\frac{3(y_1-1)^2}{r_1^5} + \frac{3\lambda y_1^2}{r_2^5} - \frac{1}{r_1^3} - \frac{\lambda}{r_2^3}\right\} + 3y_3^2\\
&\quad\left\{\frac{(y_1-1)}{r_1^5} + \frac{\lambda y_1}{r_2^5}\right\} + 3y_3^2\left\{\frac{(y_1-1)}{r_1^5} + \frac{\lambda y_1}{r_2^5}\right\} - \frac{(y_1-1)}{r_1^3} - \frac{\lambda y_1}{r_2^3} + \frac{(1-\mu)(y_1-1)}{r_1^3}\\
&+\frac{\mu y_1}{r_2^3} - x_4\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} - (x_1+\mu-1)\left\{\frac{3(x_1-1)^2}{r_1^5} + \frac{3\lambda x_1^2}{r_2^5} - \frac{1}{r_1^3} - \frac{\lambda}{r_2^3}\right\}\\
&-3x_3^2\left\{\frac{(x_1-1)}{r_1^5} + + \frac{\lambda x_1}{r_2^5}\right\} + \frac{(x_1-1)}{r_1^3} + \frac{\lambda x_1}{r_2^3} - \frac{(1-\mu)(x_1-1)}{r_1^3} - \frac{\mu x_1}{r_2^3} + v_2(t)\\
&u_3(t) = v_3(t)\\
&u_4(t) = -y_2\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} + 3y_3(y_1+\mu-1)\left\{\frac{(y_1-1)}{r_1^5} + \frac{\lambda y_1}{r_2^5}\right\} + 3y_3^3\left\{\frac{1}{r_1^5} + + \frac{\lambda}{r_2^5}\right\} -\\
&2y_3\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} + \frac{y_3(1-\mu)}{r_1^3} + \frac{\mu y_3}{r_2^3} - x_2\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} - 3x_3(x_1+\mu-1)\left\{\frac{(x_1-1)}{r_1^5} + \frac{\lambda x_1}{r_2^5}\right\} -\\
&3x_3^3\left\{\frac{1}{r_1^5} + + \frac{\lambda}{r_2^5}\right\} + 2x_3\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\} - \frac{x_3(1-\mu)}{r_1^3} - \frac{\mu x_3}{r_2^3} + v_4(t)
\end{aligned}\right\} \quad (12)$$

The new error system can be expressed as:

$$\left.\begin{aligned}
\dot{e}_1 &= e_2 + v_1(t)\\
\dot{e}_2 &= 2e_4 + e_1 + v_2(t)\\
\dot{e}_3 &= e_4 + v_3(t)\\
\dot{e}_4 &= -2e_2 + e_3 + v_4(t)
\end{aligned}\right\} \quad (13)$$

The error system (13) to be controlled is a linear system with a control input $v_i(t)$ ($i = 1, \dots 4$) as function of the error states e_i ($i = 1, \dots 4$). As long as these feedbacks stabilize the system e_i ($i = 1, \dots 4$) converge to zero as time t tends to infinity. This implies that master and the slave system are synchronized with active control. There are many possible choice for the control $v_i(t)$ ($i = 1, \dots 4$). We choose.

$$\begin{bmatrix} v_1(t)\\ v_2(t)\\ v_3(t)\\ v_4(t)\end{bmatrix} = A\begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4\end{bmatrix} \quad (14)$$

Here A is a 4×4 constant matrix to be determined. As per Lyapunov stability theory and Routh-Hurwitz criterion, in order to make the closed loop system (14) stable, proper choice of elements of A has to be made so that the system (14) must have all eigen values with negative real parts. Choosing

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -1 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \\ 0 & 2 & -1 & -1 \end{bmatrix} \quad (15)$$

and, defining a matrix B as

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \\ \dot{e_4} \end{bmatrix} = B \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad (16)$$

Where B is

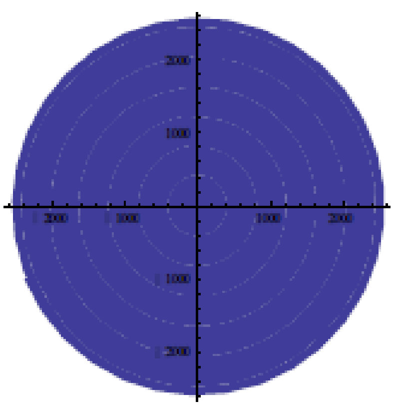
$$B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (17)$$

Clearly, B has eigen values with negative real parts. This implies $\lim_{t\to\infty} |e_i| = 0$; $i = 1, 2, 3, 4$ and hence, complete synchronization is achieved between the master and slave systems (9) and (10) respectively.

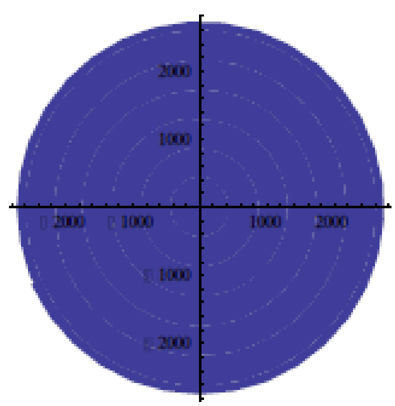
Numerical Simulation

We select the parameters $\mu = 0.001$ and $\lambda = 5$ with the initial conditions for master and slave systems $[x1(0) = 2.5,\ x2(0) = 0.4,\ x3(0) = -3.5,\ x4(0) = 0.2]$ and $[y1(0) = 5.5,\ y2(0) = 1.4,\ y3(0) = -4.5,\ y4(0) = 0.8]$ respectively. We have simulated the system under consideration using Mathematica Phase portraits and analysis of master and slave system are the witness of irregular behavior of the system (figures 2 and 3).

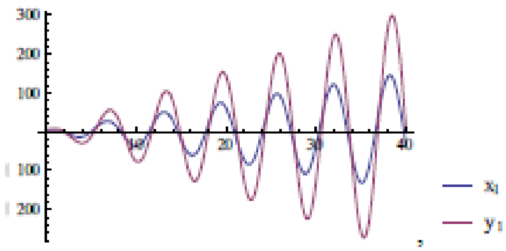
Simulation results for uncoupled system are presented in figures.4,5.6. and that of controlled system are shown in figures.7,8 and 9 respectively.



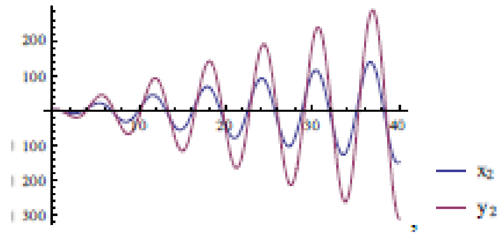
Phase portrait of master system Fig(2)



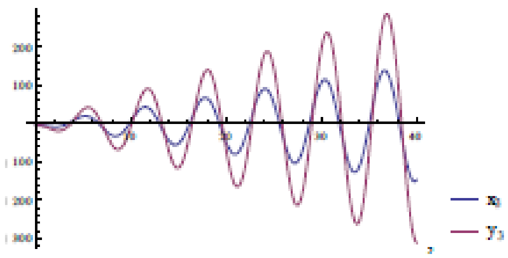
Phase portrait of slave system Fig(3)



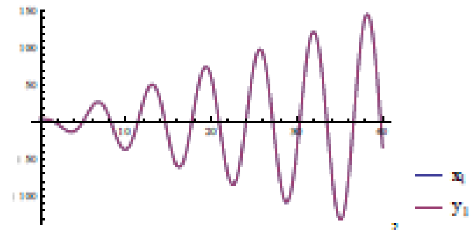
Fig(4)



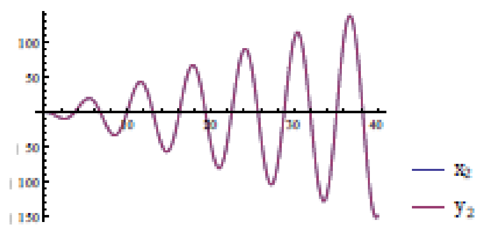
Fig(5)



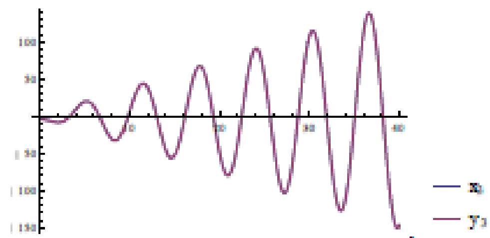
Fig(6)



Fig(7)

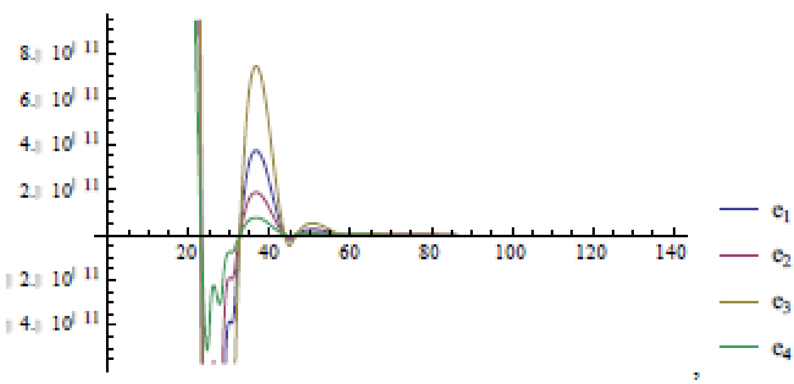


Fig(8)



Fig(9)

And for [$e1(0) = 1$, $e2(0) = 0.5$, $e3(0) = 2$, $e4(0) = 0.2$]convergence diagrams of errors are the witness of achieving synchronization between master and slave system (figure, 10).



Fig(10)

Conclusion

An investigation on synchronization in the planar magnetic-binaries problem including the effect of the gravitational forces of the primaries on the small body and transferring the origin of the coordinate system to the second mass via active control technique based on Lyapunov stability theory and Routh-Hurwitz criteria have been made. We observed that the system under consideration is chaotic for μ = 0.001 and λ = 5. Here two systems (master and slave) are synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for full chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

"Compliance with Ethical Standards"

Conflict of Interest: The authors declare that they have no conflict of interest.

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