

Economic forecasting with the help of linear production function complex variable

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Abstract

In this article one of the important problems of the base to modern theory to functions complex variable and determinations of the possibility of their using in economy is considered.

Keywords: Economic forecasting, complex variable

Introduction

The modelling of economic dynamics today is one of major and most advanced directions of modern economic science. These days huge set of models of economic dynamics is constructed, in each of these or other restrictions on initial variable are offered, the original aggregated models of interrelation between initial variable and results are under construction and offered the production functions of a different sort.

Methodology

We shall build the models of economic dynamics of the Khorezm area of the Republic of Uzbekistan for 2004 - 2015. The initial data assembled on the data of Government Statistical committee of Uzbekistan, are given in the table - 1.

To construct model of economic dynamics, these data were given in dimensionless sizes.

Attempt to construct Cobb-Douglas production function on these data has appeared senseless, as the model, which factors were found with the help of a method of the least squares (MLS), looks like:

$$Q_t = 0,923 K_t^{2,032} L_t^{-1,032} \quad (1)$$

but factors degree to Cobb-Douglas function, as is well known, lies within from zero up to unit. So it cannot be applicable for analysis economic dynamic in the given production function.

The Simple sedate production function, factors which were also found by help of MLS on non-dimensional table data 1, has such kind of view:

$$Q_t = 0,67 K_t^{2,332} L_t^{2,211} \quad (2)$$

Hence, the elasticity of capital resources use for 2004 - 2015 in Uzbekistan has made 2,332 units, and elasticity of manpower use – 2,211 units.

Table 1: The data of economic development of Khorezm region economy of Uzbekistan.

Year	Investment mil. sum	Gross regional product, mil. sum	Fixed capital stock, mil. sum	Numbers engaged thous. forehead
2004	123 802,60	483,20	930449,0	506,60
2005	63 263,50	562,00	1010865,4	522,30
2006	113 406,30	930,20	1091063,0	538,00
2007	114 503,20	1 003,70	1230510,0	547,00
2009	1823321,5	1547,80	972985,2	606,70
2010	3903000,0	1944,10	1102883,0	637,50
2011	5143000,0	2306530,3	1257135,7	625,2
2012	6574000,0	2945806,2	1366520,1	643,4
2013	10833000,0	3698000,5	1478698,7	657,2
2014	14660000,5	5061000,0	1985432,8	657,8
2015	13970000,0	56926000,0	2354788,4	659,6

Source: a statistical year-book of Uzbekistan – 2015.

According to the earlier received

$$\Delta QR > \Delta RQ \rightarrow \frac{\Delta Q}{\Delta R} \frac{R}{Q} = \varepsilon_R > 1$$

results, it tells us about that capacities of economy of the Khorezm region is unutilized, therefore, the increase of any resource - capital or labor will result in essential growth of production efficiency.

For construction of model of economic dynamics it is not enough to have production function, it is necessary to describe other interrelations of this model. As Gross regional product (GRP) is divided into consumption and savings, on the data of table-1, is it necessary to calculate norm of savings ρ that reflects part of GRP, which goes on the investment. This share has appeared extremely small - only 0,2! That is the size of the investments per year T is defined with the help of this norm of savings so:

$$I_t = 0,2Q_t \quad (3)$$

Investments promote increase of the production capital funds of the next year K_{t+1} and are expressed through outdated funds K_t with account of the share left for year main production assets μ . At calculation of this share we have met with unexpectedness, as the share of leaving of a

fixed capital has appeared... negative and is equal -0,35. That is, the funds do not leave, and on the contrary, will expand and a gain this essentially exceeds size of the investments. This paradoxical result can be explained (excepting possibility of statistical invalidation of the data of state statistics).

After announcement by former USSR independence and breakup as result to this independence of the main economic relationships between enterprises of the republics, many productions ceased to exist, or worked with very small loading the production assets. In these condition part capital assets was derived from statistical balance, as unused in economic purpose, for instance, vacant premises and heavy tool. The small investments in such capital assets promoted their "reanimation" and so increase of the cost of the capital assets turned out to be above, than investments in the capital assets.

In view of this dynamics of a capital resource for the considered period is described by the following model:

$$K_{t+1} = 1,63 K_t + I_t \quad (4)$$

The number engaged in economy has not brought similar surprises. The gain engaged has made 3 % per year ($v = 0,03$), therefore

number engaged in economy L_{t+1} is determined through number engaged in the current year L_t in view of annual rate of a gain of number engaged (v) So:

$$L_{t+1} = 1,03L_t \quad (5)$$

The equations (2) - (5) also make model of economic dynamics for the Khorezm region of Uzbekistan. However attempt to use this model for forecasting economic development of area has failed. As the table - 2 this model is visible from the data simulates unprecedented growth of volumes of manufacture. It is obvious that any of areas by such rates cannot develop that why the model comprises few mistakes.

The rapid growth of the gross regional product in models is caused that that factor degree capital resource in production function (2) is more than two. This means that increase the capital resource, for instance, in two will times bring about growing of the production result in four times more then. Hence, in spite of the fact that the model of production function not bad describes last dynamics, it is impossible to use it in model of economic dynamics of the Khorezm region.

In a line of production functions valid variable the production function in the additive form is known also. It is rather simple and has no those by remarkable theoretically deduced properties, which are inherent in power-mode production function. As the basic task of our research is the construction of model of economic dynamics of the Khorezm region to neglect the possibility of the use to this model has no sense, even the most popular production functions have demonstrated its unacceptability.

The simple linear model to production function in additive form will be of the form of:

$$Q_t = a + bK_t + cL_t \quad (6)$$

The method of the least squares allows estimating factors of this model on the data of the Khorezm region as follows:

$$Q_t = 4,2312K_t + 11,003L_t - 12,234 \quad (7)$$

Using this model and found earlier correlations (3)-(5), possible use their together as model economic dynamics and execute the forecast economic dynamics Khorezm region on prospect. The results of this forecast were provided in table-3.

Table 2: Path of the economic development Khorezm region on models with power-mode production function.

Year	Investment, It	Value of capital assets, Kt	Number engaged, Lt	Gross regional product, Qt
2016	0,562106	2,62	1,262308	14,55265
2017	0,955558	2,71	1,37554	26,88896
2018	3,102306	4,69	1,413305	87,55764
2019	35,31417	6,83	1,539571	858,8542
2020	42109,02	42,20	1,66362	1057726

Table 3: Path of the economic development of Khorezm region on models with linear production function.

year	I_t	K_t	L_t	Q_t
2016	0,380144	2,015321	1,33570	8,0502793
2017	0,402906	2,054006	1,37527	8,579732
2018	0,468277	2,198567	1,41600	9,566774
2019	0,486371	2,376932	1,45794	10,98875
2020	0,5087402	2,446432	1,50113	11,79506

It is necessary to recognize the received settlement meanings plausible, as the economic growth described in the table, is quite stacked in frameworks of possible path. Thereby, simple linear model to production function turns out to be in considered event more suitable to modeling economic dynamics, more than complex nonlinear models.

Besides production functions valid variable in models of economic dynamics can be used and production functions complex variable. From possible variety of these functions we shall estimate an opportunity of use of production function complex variable with the valid factors²:

$$I_t + iC_t = a(K_t + iL_t)^b \tag{8}$$

About possibility of the use of this type to models to production function in models economic dynamics possible to judge that its factors are changed at time, unlike power-mode production function real variable for point estimation factor which necessary to have an observations for three temporary moments, factors in models (6) can easily found. Really, taking logarithm both left and right parts (6), for instance on natural base, we shall get:

$$\ln(I_t + iC_t) = \ln a + b \ln(K_t + iL_t) \tag{9}$$

Or remember the characteristic of the logarithm of the complex number:

$$\ln R_{nt} + \varphi_i = \ln a + b \ln R_{pt} + i b \theta_t \tag{10}$$

here, $R_{nt} = \sqrt{I_t^2 + C_t^2}$, $R_{pt} = \sqrt{K_t^2 + L_t^2}$,

$$\varphi_i = \text{arctg} \frac{C_t}{I_t} \quad \theta_t = \text{arctg} \frac{L_t}{K_t}$$

As two complex numbers are equal to each other only when the valid and imaginary parts of these numbers are equal to each other, it turns out, that the equality (10) is equivalent to system of equality:

$$\begin{cases} \ln R_{nt} = \ln a_0 + b \ln R_{pt}, \\ \varphi_i = b \theta_t. \end{cases} \tag{11}$$

From second equality, it is easy to find for each t size of coefficient b, and from first, knowing meaning of coefficient to find coefficient a. Analyzing change in time of these coefficients; it is possible to accept the decision on suitability or unfitness of model to real use.

If the coefficients will not undergo essential changes in time, and will remain approximately constant, model (6) can be used for modeling real process.

It is easy to be convinced that each of coefficients complex valued models varies in time. Thus the change of each factor does not carry casual character, and on the contrary, has the brightly expressed regular component. It tells that the model of production function complex variable cannot be used for the decision of the given task. Let's consider the possibility of the use logarithmic complex valued to functions of the type³:

$$I_t + iC_t = (a_0 + ia_1) + (b_0 + ib_1) \ln(K_t + iL_t) \tag{12}$$

And again for this model, as well as for the majority of models complex variable, there is an opportunity not only to calculate coefficients of models with the help (MLS), but also to estimate meanings of each coefficient for each moment of supervision,

that is conclusive advantage of complex valued production function. If conduct preliminary central source variable production result comparatively their average arithmetical, that possible get such system two equations with two unknown coefficients:

$$\begin{cases} I'_t = b_0 \ln R_t - b_1 \varphi_t \\ C'_t = b_1 \ln R_t + b_0 \varphi_t \end{cases} \quad (13)$$

As it is seen, solving this equation, possible get the estimation of the complex factor to proportions to models (12) at each moment of the observation.

Moreover, it is possible to be convinced that the factors of model vary regularly; hence, logarithmic complex valued production function cannot be used for modeling economic dynamics of Khorezm region.

As well as in a case with production functions valid variable, let's see, whether the function will be suitable for modeling Khorezm region linear complex valued. With reference to considered case it has the following kind:

$$I_t + iC_t = (a_0 + ia_1) + (b_0 + ib_1)(K_t + iL_t) \quad (14)$$

Additionally, we shall consider an opportunity of application of this function for the description of dynamics of Khorezm region economy of the Republic of Uzbekistan. For this purpose, we shall consider dynamics of complex coefficient of proportionality on an initial interval of time. If use central variable, these coefficients are easily found from decision of system of the equations:

$$\begin{cases} I'_t = b_0 K'_t - b_1 L'_t \\ C'_t = b_0 L'_t + b_1 K'_t \end{cases} \quad (15)$$

So the observations easy to calculate the corresponding pair coefficient for each year. It is possible to notice, that both coefficient b_0 , and coefficient b_1 do not remain constant.

However, neither this, nor other coefficients have no the brightly expressed tendency to growth either fall, or any other tendency. It means that linear complex valued production function can be used for account of economic dynamics of Khorezm region. Using MLS with reference to this model [4], we shall get such kind of model:

$$I_t + iC_t = (+i) + (+i)(K_t + iL_t) \quad (16)$$

Thus, we came to a conclusion that from all variety of production functions valid and complex variable most suitable for modeling economic dynamics of Khorezm region are the linear functions. It is natural, that this conclusion cannot be distributed to all other cases; it can only be applied for the given area and in a considered interval of time. As in our disposal have two linear models of production function (valid and complex variable), we compare results how they simulate dynamics of development of Khorezm region on prospect to make a choice of the best model of these two.

The first model of economic dynamics of Khorezm region represents model of the valid variable following kind⁴:

$$Q_t = 4,2312K_t + 11,003L_t - 12,234 \quad (17)$$

$$I_t = 0,2Q_t \quad (18)$$

$$K_{t+1} = 1,63 K_t + I_t \quad (19)$$

$$L_{t+1} = 1,03L_t \quad (20)$$

Other model of dynamics of Khorezm region is a model of economic dynamics with complex valued by linear production function. As the production function (16) allows calculating division of a gross product into consumption and savings, the model has such kind:

$$I_t + iC_t = (3,34 + i3,5) + (1,53 + i4,32)(K_t + iL_t) \quad (21)$$

$$K_{t+1} = 1,63 K_t + I_t \quad (22)$$

$$L_{t+1} = 1,03L_t \quad (23)$$

Table 4: Comparative paths of the economic development of Khorezm region on different models of economic dynamics.

Year	Valid variable model			Complex variable model		
	I_t	K_t	Q_t	I_t	K_t	Q_t
2016	0,354766	2,012325	8,022745	0,34	3,00	7,6542
2017	0,402403	2,034002	8,564754	0,09	3,08	8,6012
2018	0,442234	2,238514	8,723446	- 0,25	2,95	5,3403
2019	0,474391	2,367934	9,203323	- 0,68	1,52	2,0268
2020	0,499865	2,536233	9,985623			

Results

Let's reduce results of accounts of economic dynamics of Khorezm region on prospect till 2018 on one table. But beforehand necessary to notice, that amount of two variable production results in the left part of equality (21) gives us value of a gross regional product:

$$Q_t = I_t + C_t \quad (24)$$

which is included by us in table-4.

The comparative analysis of path of development of Khorezm region received by model with production function valid variable (17) - (20) and model with production function complex variable (21) - (23) shows the following.

The model with valid variable (17) - (20) demonstrates the sure growth of a gross regional product with an insignificant nonlinear gain for an interval between 2017 and 2016 it makes 0,54, and for an interval between 2019 and 2020 makes already 0,78. Such results are easily explained that model continues the tendencies existing in 2004 - 2015, including growth of the capital.

On the contrary, the model of economic dynamics with linear production function complex valued (21) - (23) calculates a path of reduction of volume of a gross regional product (GRP). If in 2020 this model calculates GRP in from 2004 taken for base calculates for 2020.

With account that that modern world economic crisis brings about reduction volume of production and volume of the gross products of all countries and

practically all region, that model economic dynamics with production function complex valued more exactly describes the real situation, than model with production function complex variable.

Discussion and Outlook

To use any model of economic dynamics for the purposes of socio-economic forecasting, certainly, it is necessary to use adaptive methods, which allow changing coefficients of model of economic dynamics that they took into account the current information in the greater degree, than previous, so they adapted models for the changes in the tendencies of development. But it is task for other researches. The given research has shown, that the production functions complex variable not only can be used as the basic equations in models of economic dynamics, but also in some cases with their help this dynamics is simulated better, rather than with the help of models valid variable.

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