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Characterisation of Nano generalized β closed sets in Nano topological spaces

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Abstract

The objective of this paper is to introduce a new class of sets called Nano generalized β closed sets in Nano topological spaces and also investigate the characteristic of these defined sets.

Keywords: Nano β interior, Nano β closure and Nano generalized β closed sets

1. Introduction

Levine [7] introduced generalized closed sets as a generalization of closed sets in topological spaces. Andrijevic [1] introduced a new class of generalized open sets in a topological space, so called semi pre open sets, later Dontchev [4] studied generalized semi pre open sets and the equivalent notion of Nano β open sets was discussed by Gnanambal [5]. The notion of Nano topology is introduced by Lellis Thivagar [6] which is defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and he also defined Nano closed set, Nano interior and Nano closure.

The aim of this paper is to continue the study of nano generalized closed sets in nano topological space. In particular, we introduce a new class of nano sets on Nano topological spaces called Nano generalized β closed sets and obtain their characteristics with counter examples.

2. Preliminaries

Definition: 2.1[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space.

Let $X \subseteq U$. Then,

- The lower approximation of X with respect to R is the set of all objects which can be for certainly classified as X with respect to R and is denoted by $L_R(X)$.

  $L_R(X) = \cup\{R(x) : R(x) \subseteq X, x \in U\}$

  where $R(x)$ denotes the equivalence class determined by $x \in U$.
- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X) = \cap\{R(x) : R(x) \cap X \neq \phi, x \in U\}$

- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not $-X$ with respect to R and is denoted by $B_R(X)$.

  $B_R(X) = U_R(X) - L_R(X)$

Property: 2.2 [6] If (U,R) is an approximation space and $X, Y \subseteq U$, then

1) $L_R(X) \subseteq X \subseteq U_R(X)$
2) $L_R(\phi) = U_R(\phi) = \phi$
3) $L_R(U) = U_R(U) = U$
4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
10) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
11) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition: 2.3[6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.2 $\tau_R(X)$ satisfies the following axioms:

i) U and $\phi \in \tau_R(X)$.
ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X, $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets with $[\tau_R(X)]^C$ being called dual Nano topology of $\tau_R(X)$.

Remark: 2.4[6] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition: 2.5[6] If $(U, \tau_R(X))$ is a Nano topological space with respect X where $X \subseteq U$ and if $A \subseteq U$, then

i)The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by $N\operatorname{int}(A)$. $N\operatorname{int}(A)$ isthe largest Nano open subset of A.

ii)The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest Nano closed set containing A.

Definition: 2.6 [2][5][7] Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

(i) Nano semi open if $A \subseteq Ncl(N\operatorname{int}(A))$
(ii) Nano pre open if $A \subseteq N\operatorname{int}(Ncl(A))$
(iii) Nano α open if $A \subseteq N\operatorname{int}[Ncl(N\operatorname{int}(A))]$
(iv) Nano regular open if $A = N\operatorname{int}(Ncl(A))$
(v) Nano b open if $A \subseteq Ncl(N\operatorname{int}(A)) \cup N\operatorname{int}(Ncl(A))$
(vi) Nano β open(Nano semi-pre open) if $A \subseteq Ncl[N\operatorname{int}(Ncl(A))]$

$NSO(U,X)$, $NPO(U,X)$, $N\alpha O(U,X)$, $NRO(U,X)$, $NBO(U,X)$ and $N\beta O(U,X)$ respectively, denote the families of all Nano semi open, Nano pre open, Nano α open, Nano regular open, Nano b open and Nano β open subsets of U.

Let $(U,\tau_R(X))$ be a Nano topological space and $A\subseteq U$. Then A is said to be Nano semi closed, Nano pre closed, Nano α closed, Nano regular closed, Nano b closed and Nano β closed if its complement is respectively Nano semi open, Nano pre open, Nano α open, Nano regular open, Nano b open and Nano β open.

Definition: 2.7 [3] A subset A of $(U,\tau_R(X))$ is called Nano generalized closed set (briefly Ng closed) if $Ncl(A)\subseteq V$ whenever $A\subseteq V$ and V is Nano open in $(U,\tau_R(X))$.

3. Nanogeneralized β closed sets

Definition: 3.1 If $(U,\tau_R(X))$ is a Nano topological space with respect X where $X\subseteq U$ and if $A\subseteq U$, then

i)The Nano β interior of a set A is defined as the union of all Nano β open subsets contained in A and is denoted by $N\beta\operatorname{int}(A)$. $N\beta\operatorname{int}(A)$ is the largest Nano β open subset of A.

ii)The Nano β closure of a set A is defined as the intersection of all Nano β closed sets containing A and is denoted by $N\beta cl(A)$. $N\beta cl(A)$ is the smallest Nano β closed set containing A

Definition: 3.2 A subset A of Nano topological space $(U,\tau_R(X))$ is called Nano generalized β closed set (briefly Ngβ closed) if $N\beta cl(A)\subseteq V$ whenever $A\subseteq V$ and V is Nano open in $(U,\tau_R(X))$.

Theorem: 3.3 Every Nano β closed set in $(U,\tau_R(X))$ is Nano gβ closed set in $(U,\tau_R(X))$.

Proof: Assume that A is a Nano β closed set in $(U,\tau_R(X))$ and let V is Nano open in $(U,\tau_R(X))$ such that $A\subseteq V$, $N\beta cl(A)=A\subseteq V$. That is $N\beta cl(A)\subseteq V$. Therefore A is Nano g β closed set.

Remark: 3.4 The converse of the above theorem need not be true which can be seen from the following example.

Example: 3.5 Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b\},\{c\},\{d\}\}$ and $X=\{a,d\}$. Then the Nano topology is defined as $\tau_R(X)=\{U,\phi,\{a,d\}\}$. Here the set $\{a,c,d\}$ is Nano g β closed but not Nano β closed in U.

The following theorem can also be proved in a similar way.

Theorem: 3.6 Let $(U,\tau_R(X))$ be a Nano topological space and $A\subseteq U$. Then

(i) Every Nano closed set is Nano g β closed set.
(ii) Every Nano semi closed set is Nano g β closed set.
(iii) Every Nano pre closed set is Nano g β closed set.
(iv) Every Nano α closed set is Nano g β closed set.
(v) Every Nano regular closed set is Nano g β closed set.
(vi) Every Nano b closed set is Nano g β closed set.
(vii) Every Nano g closed set is Nano g β closed set.
(viii) Every Nano gs closed set is Nano g β closed set.
(ix) Every Nano α g closed set is Nano g β closed set.
(x) Every Nano g r closed set is Nano g β closed set.

Remark: 3.7 Reverse implications of the above theorem 3.6 need not be true which can be seen from the following example.

Example: 3.8 Let $U = \{a, b, c, d, e\}$ with $U/R = \{\{a\}, \{b, c, d\}, \{e\}\}$ and $X = \{a, b\}$. Then the Nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, c, d\}, \{b, c, d\}\}$. Here the set $\{a, b, c\}$ is Nano g β closed but not Nano closed, Nano semi closed, Nano pre closed, Nano α closed, Nano regular closed, Nano b closed, Nano g closed, Nano g s closed, Nano α g closed, Nano g r closed in U.

4. Characterisation of nano generalized β closed sets

Theorem: 4.1 The union of two Nano g β closed sets need not be Nano g β closed set which can be seen from the following example.

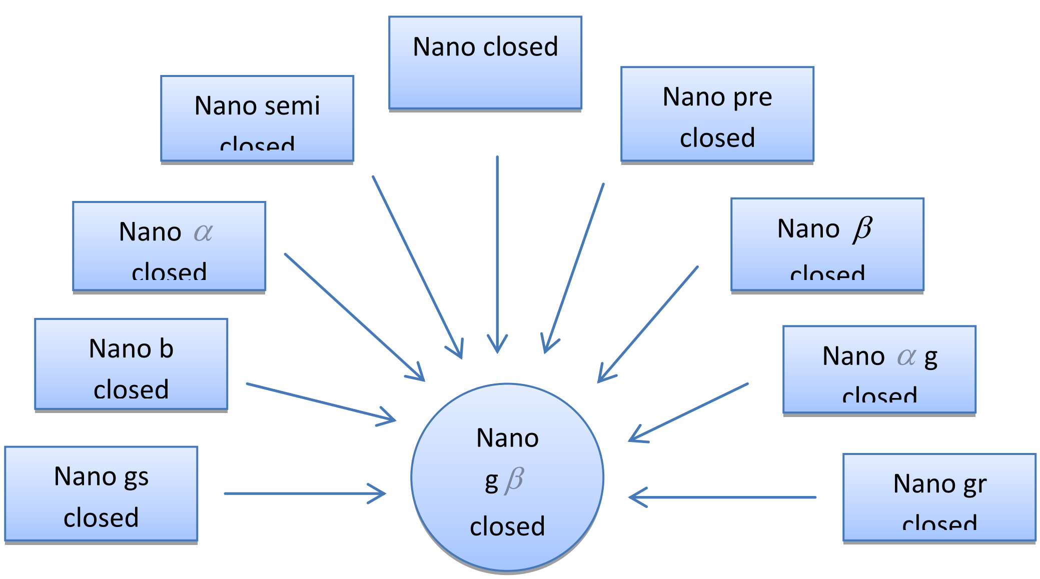
Example: 4.2 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the Nano topology is defined as $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Here the sets $\{a\}$ and $\{b, d\}$ are Nano g β closed sets but $\{a\} \cup \{b, d\} = \{a, b, d\}$ is not Nano g β closed set in U.

Theorem: 4.3 The intersection of two Nano g β closed sets need not be Nano g β closed set which can be seen from the following example.

Example: 4.4 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c\}, \{d\}\}$ and $X = \{a, d\}$. Then the Nano topology is defined as $\tau_R(X) = \{U, \phi, \{a, d\}\}$. Here the sets $\{a, b.d\}$ and $\{a, c, d\}$ are Nano g β closed sets but $\{a, b, d\} \cap \{a, c, d\} = \{a, d\}$ is not Nano g β closed set in U.

Theorem: 4.5 Let A be a $Ng\beta$ closed set in $(U, \tau_R(X))$ then $N\beta cl(A) - A$ does not contain any non-empty Nano closed set in $(U, \tau_R(X))$.

We have the following implications for the properties of subsets:



Proof: Let A be a $Ng\beta$ closed set in $(U,\tau_R(X))$ and F be Nano closed subsets of $N\beta cl(A)-A$. That is $F\subseteq N\beta cl(A)-A$ implies $F\subseteq N\beta cl(A)\cap A^C$. That is $F\subseteq N\beta cl(A)$ and $F\subseteq A^C$ which implies $A\subseteq F^C$ where F^C is a Nano open set. Since A is Nano gβ closed, $N\beta cl(A)\subseteq F^C$. That is $F\subseteq (N\beta cl(A))^C$.
Thus
$F\subseteq N\beta cl(A)\cap (N\beta cl(A))^C=\phi, F=\phi$.
Hence $N\beta cl(A)-A$ does not contain any non-empty Nano closed set in $(U,\tau_R(X))$.

Theorem: 4.6 Let A be a $Ng\beta$ closed set in $(U,\tau_R(X))$ then A is Nano β closed if and only if $N\beta cl(A)-A$ is Nano closed set in $(U,\tau_R(X))$.

Proof: Let A be a $Ng\beta$ closed set. Assume that A is Nano β closed then we have $N\beta cl(A)=A$, $N\beta cl(A)-A=\phi$.
Hence $N\beta cl(A)-A$ is Nano closed. Conversely, assume that $N\beta cl(A)-A$ be Nano closed. Then by theorem 4.5 $N\beta cl(A)-A$ does not contain any non-empty closed set. Thus $N\beta cl(A)-A=\phi$. That is $N\beta cl(A)=A$. Therefore is Nano β closed.

Theorem: 4.7 If A is Nano gβ closed in $(U,\tau_R(X))$ and B is any set in $(U,\tau_R(X))$ such that $A\subseteq B\subseteq N\beta cl(A)$, then B is also Nano gβ closed in $(U,\tau_R(X))$.

Proof: Let A be a $Ng\beta$ closed set of U such that $A\subseteq B\subseteq N\beta cl(A)$. Let $B\subseteq V$ where V be Nano open set in U then $A\subseteq V$. Since A is $Ng\beta$ closed, we have $N\beta cl(A)\subseteq V$. Now $B\subseteq N\beta cl(A)$, $N\beta cl(B)\subseteq N\beta cl(N\beta cl(A))=N\beta cl(A)\subseteq V$, $N\beta cl(B)\subseteq V$. Therefore B is Nano gβ closed set in U.

Theorem: 4.8 Let A be Nano open and Nano gβ closed set in $(U,\tau_R(X))$ then $A\cap F$ is Nano g β closed whenever $F\in N\beta cl(U,X)$.

Proof: Let A be Nano open and Nano gβ closed set then $N\beta cl(A)\subseteq A$ and $A\subseteq N\beta cl(A)$. Therefore $N\beta cl(A)=A$. Thus A is Nano β closed. Hence $A\cap F$ is Nano β closed in U which implies that $A\cap F$ is Nano g β closed in U.

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