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Information Aggregation and Option Pricing: An Empirical Comparison of EWMA and Black and Scholes Models

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Abstract

Volatility is the most important concept in options trading. No one can consistently predict the market, not even the experts. Yet many investors think they can guess what will happen, based on hunches or rumors. Unless we know precisely when to buy or sell, we can, and probably will miss the market. This can really cost us. Most of the market's gain and loss occur in just a few, but unpredictable, trading days. In this paper we study the differences in unpredictability of stock and option prices using Black and Scholes model and EWMA volatility model. This difference eventually affects options range and premiums.

Keywords: Black and Scholes model, Option pricing, EWMA volatility, Implied volatility, Calibration

Introduction

Investing in market, whether stock market or derivatives market, involves risk. The value of our investment will fluctuate over time and we may gain or lose money. Volatility plays a significant role in the pricing derivatives (e.g. options).Investors who are trading in stock market, having a little technical knowledge about the behavior of the option market, may think about investing in option market and want to make profit out of it. If they want to make an assumption of market volatility which is one of the key aspects in options trading, they can use EWMA volatility which can be calculated only using stock index data, as a proxy of implied volatility. We choose benchmark Black and Scholes model to replicate derivatives market, as volatility is the only variable in the Black-Scholes model that is unobservable. Moreover the effective characterization of volatilityhas the potential to address the contagion effects in the market which is caused by "transmission" of volatility from one country to another [1] ; and the stock market and the derivative markets might influence the contagion effect differently. Volatility can explain extreme events as Blake [2] explains that the October 1987 crash could have resulted from volatility changes; however stock induced extremity and derivatives induced extremity might have different effect in such crashes. Shiller [3] argues the market's volatility dynamics can be applied to macroeconomic variables, particularly as the stock market is a well-known leading indicator of the economy. Shiller [3] also claims volatility can be used as a measure of market efficiency.Options have become important to industry, particularly as they can be used to hedge risk [7]. In fact in many situations it is more attractive to speculators and hedgers to trade an option rather than an underlying due to the limited loss. However option induced volatility could induce these market features in different ways because usually the quantification of volatility from derivative information content and that that from stock information content do differ significantly.

Log-normal Properties of Stock Prices

The model of stock price behavior used by Black-Scholes model [4]. It assumes that percentage changes in the stock price in a short period of time are normally distributed. Define:

μ: Expected return on stock per year

σ : Volatility of the stock price per year

The mean of the return in time Δtis : μ Δt and the standard derivation of the return is : $\sigma\sqrt{\Delta t}$, so that

$$\frac{\Delta S}{S} \sim \phi\left(\mu\,\Delta \mathrm{t}, \sigma^2 \Delta t\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Where ΔS is the change in the stock price S in time Δ t, and $\phi(m,v)$ denotes a normal distribution with mean *m* and variance *v* [6].

Hence the model implies,

$$\ln S_T - \ln S_0 \sim \phi\left[(\mu - \frac{\sigma^2}{2})\mathrm{T},\ \sigma^2 T\right]$$

$$\Rightarrow \ln \frac{S_T}{S_0} \sim \phi\left[(\mu - \frac{\sigma^2}{2})\mathrm{T},\ \sigma^2 T\right]$$

$$\Rightarrow \ln S_T \sim \phi\left[\ln S_0 + (\mu - \frac{\sigma^2}{2})\mathrm{T},\ \sigma^2 T\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

where, S_T is the stock price at a future time T and S_0 is the stock price at time 0. Equation (2) shows that ln S_T is normally distributed, so that S_T has a log-normal distribution. The mean of ln S_T is $\ln S_0 + (\mu - \frac{\sigma^2}{2})\mathrm{T}$ and the standard deviation is $\sigma\sqrt{T}$ [5]

Black–Scholes Option Pricing Formula

The Black–Scholes formula calculates the price of European put and call options. This price is consistent with the Black–Scholes equation [4]; this follows since the formula can be obtained by solving the equation for the corresponding terminal and boundary conditions.

The values of call and put option for a non-dividend paying underlying stock in terms of the Black–Scholes parameter are:

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}, \quad for\,Call option \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

$$P(S,t) = N(-d_2)Ke^{-r(T-t)} - N(d_1)S, \quad for\,Put option$$

The price of the corresponding put option is based on put-call parity
Where,

$$\left.\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \\ d_2 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

For both, as above:
$N(.)$: Cumulative distribution function of the standard normal distribution
T-t : Time to maturity
S : Spot price of the underlying asset
K : Strike price
σ :is the volatility of returns of the underlying asset
r : Risk free rate (annual rate)

Historic and Implied Volatility

Historical volatility (HV) is the realized volatility of a financial instrument over a given time period. Generally, this measure is calculated by determining the average deviation from the average price of a financial instrument in the given time period. Standard deviation is the most common but not the only way to calculate historical volatility. Historical Volatility reflects the past price movements of the underlying asset, while implied volatility is a measure of market expectations regarding the asset's future volatility. Historical volatility is also referred to as the asset's actual or realized volatility. There are different ways of measuring historical volatility.

The first kind of historical volatility, that is often called statistical volatility, is calculated as a standard deviation of a stock's returns over a fixed number of days. Return is defined as the natural logarithm of close-to-close prices. The section Price and Return's distribution demonstrates why using a logarithm of price relations is more convenient than using that of simple price relations. Standard deviation is a statistical measure of the variability of a set of numbers. Note, this volatility is a type of historical volatility, but not the only one. Nevertheless, when referring to historical volatility we will have in mind a standard deviation of stock's returns.

Implied volatility is the estimated volatility of a security's price. In general, implied volatility increases when the market is bearish, when investors believe that the asset's price will decline over time, and decreases when the market is bullish, when investors believe that the price will rise over time. This is due to the common belief that bearish markets are riskier than bullish markets. Implied volatility is a way of estimating the future fluctuations of a security's worth based on certain predictive factors [9].

Implied volatility is one of the deciding factors in the pricing of options. Options, which give the buyer the opportunity to buy or sell an asset at a specific price during a pre-determined period of time, have higher premiums with high levels of implied volatility, and vice versa. Implied volatility approximates the future value of an option, and the option's current value takes this into consideration. Implied volatility is an important thing for investors to pay attention to; if the price of the option rises, but the buyer owns a call price on the original, lower price, or strike price, that means he or she can pay the lower price and immediately turn the asset around and sell it at the higher price [8].

It is important to remember that implied volatility is all probability. It is only an estimate of future prices, rather than an indication of them. Even though investors take implied volatility into account when making investment decisions, and this dependence inevitably has some impact on the prices themselves, there is no guarantee that an option's price will follow the predicted pattern. However, when considering an investment, it does help to consider the actions other investors are taking in relation to the option, and implied volatility is directly correlated with market opinion, which does in turn affect option pricing.

Another important thing to note is that implied volatility does not predict the direction in which the price change will go. For example, high volatility means a large price swing, but the price could swing very high or very low or both. Low volatility means that the price likely won't make broad, unpredictable changes.

Implied volatility is the opposite of historical volatility, also known as realized volatility or statistical volatility, which measures past market changes and their actual results. It is also helpful to consider historical volatility when dealing with an option, as this can sometimes be a predictive factor in the option's future price changes.

Implied volatility also affects pricing of non-option financial instruments, such as an interest rate cap, which limits the amount by which an interest rate can be raised.

Estimating Volatility

Define σ_n as the volatility of a market variable on day n, as estimated at the end of day $n-1$.

The variance rate is the square of volatility, σ_n^2 ,on day n.

Suppose the value of the market variable at the end of the day i is S_i. The continuously compounded rate of return during day / (between end of prior day (i.e. $i-1$) and end of day i) is expressed as :

$$r_i = \ln \frac{S_i}{S_{i-1}} \qquad \text{.........................(5)}$$

Next, using the standard approach to estimate σ_n from historical data, we will use the most recent m-observations to compute an unbiased estimator of the variance:

$$\sigma_n^2 = \frac{\sum_{i=1}^{m} (r_{n-i} - \bar{r})^2}{m-1} \qquad \text{.........................(6)}$$

Where, $\bar{r}$ is the mean of r_i: $\bar{r} = \frac{\sum_{i=1}^{m} r_{n-i}}{m}$

Next, let us assume $\bar{r} = 0$ and the maximum likelihood estimate of the variance rate:

$$\sigma_n^2 = \frac{\sum_{i=1}^{m} r_{n-i}^2}{m} \quad \text{... ... ... (7)}$$

So far, we have applied equal weights to all r_{n-i}^2 , so the definition above is often referred to as the equally-weighted volatility estimate.

Earlier, we stated our objective was to estimate the current level of volatility σ_n , so it makes sense to give higher weights to recent data than to older ones. To do so, let us express the weighted variance estimate as follows:

$$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i \; r_{n-i}^2 \quad \text{... ... ...(8)}$$

Where :

- α_i is the amount of weight given to an observation *i*-days ago.
- $\alpha_i \geq 0$
- $\sum_{i=1}^{m} \alpha_i = 1$

So, to give higher weight to recent observations, $\alpha_i \geq \alpha_{i+1}$

A possible extension of the idea above is to assume there is a long-run average variance V_L **,** and that it should be given some weight:

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i \; r_{n-i}^2 \quad \text{... ... ...(9)}$$

Where :

- $\gamma + \sum_{i=1}^{m} \alpha_i = 1$
- $V_L > 0$

The model above is known as the ARCH model, proposed by Engle in1994.

$$\sigma_n^2 = \omega + \sum_{i=1}^{m} \alpha_i \; r_{n-i}^2 \quad \text{... ... ...(10)}$$

Estimating EWMA (Exponentially Weighted Moving Average) Volatility

EWMA is a special case of the equation (10). In this case, we make it so that the weights of variable α_i decrease exponentially as we move back through time.

$$\alpha_{i+1} = \lambda\alpha_i = \lambda^2\alpha_{i-1} = ... = \lambda^{n+1}\alpha_{i-n} \quad \text{... ... ...} (11)$$

Unlike the earlier presentation, the EWMA includes all prior observations, but with exponentially declining weights throughout time.

Next, we apply the sum of weights such that they equal the unity constraint:

$$\sum_{i=1}^{\infty}\alpha_i = \alpha_1\sum_{i=0}^{\infty}\lambda^i = 1 \quad \text{... ... ...} (12)$$

For $\|\lambda\| < 1$, the value of $\alpha_1 = 1 - \lambda$

Now we plug those terms back into the equation. For the σ_{n-1}^2 estimate:

$$\sigma_{n-1}^2 = \sum_{i=1}^{n-1}\alpha_i r_{n-1-i}^2 = \alpha_1 r_{n-2}^2 + \lambda\alpha_1 r_{n-3}^2 + ... + \lambda^{n-3}\alpha_1 r_1^2$$

$$\sigma_{n-1}^2 = (1-\lambda)(r_{n-2}^2 + \lambda r_{n-3}^2 + ... + \lambda^{n-3} r_1^2) \quad \text{... ... ...} (13)$$

And the σ_n^2 estimate can be expressed as follows:

$$\sigma_n^2 = (1-\lambda)(r_{n-1}^2 + \lambda\, r_{n-2}^2 + ... + \lambda^{n-2}\, r_1^2)$$

$$\sigma_n^2 = (1-\lambda)r_{n-1}^2 + \lambda(1-\lambda)(r_{n-2}^2 + \lambda\, r_{n-3}^2 + ... + \lambda^{n-3}\, r_1^2)$$

$$\sigma_n^2 = (1-\lambda)\, r_{n-1}^2 + \lambda\,\sigma_{n-1}^2 \quad \text{... ... ...} (14)$$

Now, to understand the equation better:

$$\sigma_n^2 = (1-\lambda)\, r_{n-1}^2 + \lambda\sigma_{n-1}^2$$

$$\sigma_n^2 = (1-\lambda)\, r_{n-1}^2 + \lambda((1-\lambda)\, r_{n-2}^2 + \lambda\sigma_{n-2}^2)$$

$$\sigma_n^2 = (1-\lambda)\, r_{n-1}^2 + \lambda(1-\lambda)\, r_{n-2}^2 + \lambda^2(1-\lambda)\, r_{n-3}^2 + \lambda^3\sigma_{n-3}^2)$$

$$\sigma_n^2 = (1-\lambda)(r_{n-1}^2 + \lambda r_{n-2}^2 + \lambda^2 r_{n-3}^2 + ... + \lambda^{m+1} r_{n-m}^2) + \lambda^{m+2}\sigma_{n-m}^2) \quad \text{... ... ...} (15)$$

For a larger data set, the $\lambda^{m+2}\sigma_{n-m}^2$ is sufficiently small to be ignored from the equation.

The EWMA approach has one attractive feature: it requires relatively little stored data. To update our estimate at any point, we only need a prior estimate of the variance rate and the most recent observation value.A secondary objective of EWMA is to track changes in the volatility. For small λ values, recent observations affect the estimate promptly. For λ values closer to one, the estimate changes slowly based on recent changes in the returns of the underlying variable.

Data and Calibration

We consider 4 years S&P 500 index data, starting from 25th June2004 to 25th June 2008, to estimate EWMA volatility. This is the period when markets have experienced a strong shift in volatilities. We compared the estimates over different information aggregation periods; e.g. we estimated EWMA volatilities using four years return, three years return and two years return. Then such volatility estimates are used as proxies to the sole parameter of the celebrated Black and Scholes model.

By using MATLAB function, we construct call price table for Options which is denoted as Model price. Then we found out the root mean square error (RMSE) from market price and model price.

Later we find out Black-Scholes Volatility using another MATLAB code. For this we have taken three days option data (25th june,2008, 26th june,2008 and 27th june,2008). There is 123 options in one day. We use a table where there is five different columns. They are : stock index, strike price, maturity, bid price and ask price. Using those information we find out the Black-Scholes volatility. Using this result, we find out corresponding model price for the options. Following the same process, we found out RMSE as described earlier.

In our first table, we use 4 year stock index data to find out EWMA volatility and one day option data for Black-Scholes volatility. We also show the corresponding RMSE. Here, we can easily see and understand that current information is more effective in capturing market volatility.

Table 1: Comparison of implied and historical volatilities in option pricing. EWMA volatilities are estimated from 4,3 and 2 years recent stock information; and Implied volatility is calibrated using BS model on 25th June, 2008 the day when 123 options were traded.

Time Period	Black-Scholes Volatility (RMSE)	EWMA Volatility (RMSE)
4 year	0.1218 (5.35)	0.1431 (7.5919)
3 year		0.1428 (7.5371)
2 year		0.1425 (7.4828)

In our second table, we take one more additional day for both EWMA volatility and Black-Scholes volatility. It is evident that market volatility is increasing as time progresses.

Table 2: Comparison of implied and historical volatilities in option pricing. EWMA volatilities are estimated from 4,3 and 2 years recent stock information; and Implied volatility is calibrated using BS model on 25th June, 2008 and 26th June, 2008 when 269 of options were traded.

Time Period	Black-Scholes Volatility (Error)	EWMA Volatility (Error)
4 year	0.1284 (5.599)	0.1437 (7.70)
3 year		0.1435 (7.6656)
2 year		0.1433 (7.6287)

In our third table, we take one more additional day in finding EWMA volatility and Black-Scholes volatility. We find out similar result as described earlier. Volatility is increasing as we move on.

Table 3: Comparison of implied and historical volatilities in option pricing. EWMA volatilities are estimated from 4,3 and 2 years recent stock information; and Implied volatility is calibrated using BS model on 25th June, 2008 and 26th June, 2008 and 27th when 425 of options were traded.

Time Period	Black-Scholes Volatility (RMSE)	EWMA Volatility (RMSE)
4 year	0.1307 (5.79)	0.1439 (7.74)
3 year		0.1437 (7.7028)
2 year		0.1434 (7.6471)

Table 4: Forecasting 2 days ahead pricing performance using the calibration on 25th June, 2008 for BS model; and last 4,3 and 2 years historical volatilities until 25th June, 2008 for EWMA model

Time period	Black-Scholes Volatility (RMSE)	EWMA Volatility (RMSE)
4 year	0.1278 (5.55)	0.1371 (6.5889)
3 year		0.1367 (6.5299)
2 year		0.1364 (6.4864)

Table 5 : Forecasting 1 days ahead pricing performance using the calibration on 25th June, 2008 and 26th June, 2008 for BS model; and last 4,3 and 2 years historical volatilities until 26th June, 2008 for EWMA model

Time period	Black-Scholes Volatility (RMSE)	EWMA Volatility (RMSE)
4 year	0.1358 (6.40)	0.1421 (7.41)
3 year		0.1417 (7.34)
2 year		0.1414 (7.287)

Analysis of calibration result

Conclusions

Volatility plays a significant role in the pricing of options. As traders, we must understand and pay close attention to a market's volatility as we build our option strategies. When we see extreme volatility in a market, we feel an overwhelming sense of uncertainty. The normal psyche of a trader is to react emotionally as markets make sharp moves higher and lower. This emotion and shift of sentiment has a direct relationship to the pricing of options. Traders will bid up the premium on both the call side and put side of that given market. On the flip side, we become comfortable with a market when volatility is very stable. Consistency with the market and our approach to the market creates comfort. When we see a market repeatedly bounce off of a support level or pull back from a resistance level, we feel more confident with these levels. But this paper shows that determining these levels with derivatives information and stock information can impress and upset traders with confusing hopes and despair. These levels allow traders to have a gauge on a market's trading range. Options that are outside of this range can be viewed as having a lower probability of finishing in-the-money and therefore will be assigned a smaller premium.So option premiums significantly differ based on whether traders believe in option market or stock markets fluctuations. Eventually, we conclude that EWMA volatility can be used as a guess but not as effective as implied volatility.

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