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PREDICTION OF INFLATION RATE: A KERNEL QUANTILE REGRESSION WITH TWO RADIAL BASIS FUNCTIONS

[*]Nwakuya Maureen Tobechukwu and Orumie Cynthia Ukamaka

Department of Mathematics and Statistics, University of Port Harcourt, Nigeria
*Email: maureen.nwakuya@uniport.edu.ng;
Email: ukamaka.orumie@uniport.edu.ng

ABSTRACT

This paper presented Kernel quantile regression in the prediction of inflation rate based on foreign exchange, crude oil price and currency in circulation. Two Radial basis kernel functions namely the Gaussian kernel and the Laplace kernel were applied in the analysis using four different sigma values (5,10,100,1000). Predictions at four quantiles (0.25, 0.50, 0.75 and 0.95) were made and comparison of the predicted values and the observed values were done based on the Root Mean Square Error of Prediction, Relative absolute error, Correlation and Paired T-test. Non-linearity test, normality test, Heteroscedasticity tests were carried out and the results revealed non-linearity, non-normality and presence of heteroscedasticity. The results reveal that the prediction error using the Gaussian kernel function was high for sigma 5,10,100, while that of the Laplace was only high for sigma value 5. The correlation between the observed and predicted was very high for both kernel functions at all quantiles but the paired t-test for the Gaussian kernel showed significant difference all the sigma except 1000, while there was not significant difference using the Laplace kernel at all quantiles. This paper concludes that the Laplace kernel is less sensitive to the sigma value as it makes good predictions at almost the whole considered sigma values but the Gaussian kernel is seen to have good prediction for sigma value 1000 hence it is very sensitive to the sigma value in the kernel function.

Keywords: Gaussian kernel, Laplace kernel, Kernel function, Kernel Quantile Regression and Radial Basis Kernel Function.

1. INTRODUCTION

Inflation is a major crisis in any economy in that it affects purchasing power of money. In Nigerian, inflation rate from January 2010 to June 2022 has been on serious fluctuations over the years, in June of 2022 the value hit 18.6 as stated by Nigerian Statistical bulletin. Basically inflation rate measures how prices increase over time, causing the decrease in the purchasing power of money. This study looks at the relationship between inflation rate, foreign exchange, crude oil price and currency in circulation of Nigeria from January 2010 to June 2022 in other to predict the inflation rate. Currency in circulation simply refers to notes, coins, or any other physical forms of money that are used in transactions between buyers and sellers. It affects inflation in that if the money supply increases at a faster rate than the economy's ability to produce goods and services, then inflation will result. Foreign exchange is also a major factor that affects inflation. In every economy an increase in the foreign exchange rate results to reduced price of domestic goods for foreign consumers, which leads to an increase of exports and total demands and prices. Crude oil is another vital economic factor; any increase in oil prices affects inflation, which measures the overall rate of price increases across the economy. These factors amongst others affect inflation but this study is limited to these three. In other to study this, regression analysis is most suited. Regression analysis is used for prediction and it is customarily bothered with obtaining a real-valued function f whose values f(x) corresponds to the conditional mean of y. In literature a lot of researchers have developed different techniques for this purpose, such as least mean square (LMS) regression Vinod (1978), robust regression Huber (1981) amongst others. Regression analysis comprises of both linear and non-linear estimation and these adopt different methodology.

In regression analysis, sometimes linear regression is not adequate particularly if there is a nonlinear relationship between the response variable and the predictors. In other to circumvent this situation, many nonlinear regression models have been proposed. Contrary to linear regression were usually a single constant coefficient for each predictor variable is estimated, in non-linear estimation, sometimes smoothing is involved such as kernel smoother as presented by Hastie et al., 2009. Also contrary to the linear regression the kernel regression estimates a smooth function of the predictor variables. Kernel regression mostly has more advantage over some other non-parametric methods like splines because of its ability to handle many predictor variables/high dimensional data sets quiet easily. Kernel regression has a kernel function which basically explains the measure of resemblance between each pair of data points and this measure of resemblance is employed in obtaining the weight that should be attached to each data point. In kernel regression attention is given to data points that are nearer than those far apart, this helps in the reduction of the number of parameters to be estimated and also enables in obtaining correct predictions. The bridge between linearity and non-linearity is laid out by kernel regression with step by step processes that can be conveyed with regards to dot products Kumar et al (2012).

In this paper a kernel regression model is employed to predict inflation rate from foreign exchange, crude oil price and currency in circulation in Nigeria. The prediction will be based on four different values of sigma for two different kernel functions namely the Gaussian kernel and Laplace kernel. Both functions are radial basis function and are used when this no further prior knowledge available about the data. Our aim is to research on the predictive accuracy of the two kernel functions while adjusting the value of the sigma at four different quantiles (0.25, 0.5, 0.75, 0.95). This research will ultimately aid researchers to make right decisions when applying kernel quantile regression.

1.2 **Kernel Quantile regression**:

Quantile regression was introduced by Koenker and Bassett (1978), and it is seen as an extension of the classical least squares estimation which hinges on conditional mean. Quantile regression thus extends it to the estimation of a collection of models for many conditional quantile functions. In this extension the median regression (50^{th} quantile) is a special case in that it minimizes the sum of absolute errors while the other conditional quantile functions are estimated by minimizing an off centered weighted sum of absolute errors, Koenker and Hallock (2001). The vital approach behind quantile estimation comes from the fact that the minimization of the l_1-loss function for a location estimator produces the median. In minimizing $\sum_{i=1}^{u} |y_i - \mu|$for a value of μ the terms $y_i - \mu$have to be symmetric in order for the derivative with respect to μ to disappear, Takeuchi et al (2005). This idea was expanded by Koenker (2005) to accommodate estimation of regression parameters for any quantile by simply slanting the loss function in a suitable way. Kernel quantile regression is an evolving quantile regression technique in the field of nonlinear quantile regressions as demonstrated by Takeuchi et al (2006) and Youjuan et al., (2007). Kernel quantile regression has the ability to model the nonlinear characteristics of time series data. Koenker (2005) stated in his monograph on quantile regression that Kernel Quantile Regression is more efficient in comparison to nonlinear quantile regression. Youjuanet al (2007) also carried out some work on Kernel Quantile Regression in coming up with a well structure step by step process for the computation. Kernel Quantile Regression has the advantage of using kernel functions (weighting functions) to model dependence, which allows modeling of both normal and non-normal data.

1.3 Literature review

Kernel quantile regression can be used to forecast value at risk, using past return levels as a training set (Wang et al 2009). Cattaneo et al (2019) in their paper proposed an instinctual and easily operated nonparametric density estimator that is hinged on local polynomial methods. The proposed estimator is fully adaptive to boundaries and it's automatic, it also does not need data transformation or pre-binning. Scholkopf et al. (2000) outlined the procedure for estimating quantiles non-parametrically using an extension of the ν-trick but its demerit is that the upper and lower quantiles are estimated simultaneously. Bang et al (2017) in there study came up with a new regression model named composite kernel quantile regression (CKQR). Their paper estimated robust estimation of a nonlinear regression function using the sum of multiple check functions as a loss in reproducing kernel Hilbert spaces. Lee et al (2022) developed a penalized kernel quantile regression with theoretical and computational characteristics in varying coefficient models. They demonstrated that the technique consistently identifies the partially linear structure of the varying coefficient model and also shows the same consistency with increased number of predictors with a sample size. They also developed an efficient step by step procedure that makes use of alternating direction method of multipliers that guarantees computational convergence. Mulyatia et al (2018) presented the application of kernel quantile regression in predicting extreme rainfall and they compared it to kernel quantile regression with principal components. Their findings revealed that both models gave similar predictions based on Root mean square error of prediction and correlation.

2. MATERIALS AND METHODS:

This study made use of data from Nigerian Statistics Bulletin from January 2010 to June 2022 on inflation rate, currency in circulation (CIC), foreign exchange (RE) and crude oil price (COP). Inflation rate is the response variable while CIC, RE and COP are the predictor variables. The study tries to predict inflation rate based on the predictor variables. The analysis was done using R software and excel. Linearity test, normality test, heteroscedasticity test were carried out. The response variable was confirmed to be a continuous data. In this work the predicted values were obtained using two radial Basis Kernel function namely the Gaussian and the Laplace Kernel function. Different values of sigma parameter (5, 10, 100 and 1000) in the kernel function were used. The prediction was obtained at four different quantiles 0.25, 0.50, 0.75 and 0.95. The predicted values were compared to the observed in terms of Root Mean square error of prediction (RMSEP), Relative absolute error and correlation. The paired T-test was performed to compare the similarity between the observed and the predicted. The model formulationand the comparison criteria are presented below.

2.1 **Model Formulation:**

Given a set of training data with n input variables and response variable$(X_1, Y_1), \ldots, (X_n, Y_n)$, where $X_i \epsilon \mathbb{R}^p$ are the input variable and $Y_i \epsilon \mathbb{R}$ is the response variable. As suggested by Koenker et al (1994), to obtain 100(τ)% quantile of the conditional distribution of Y/X, for any level of quantile $\tau\epsilon(0,1)$ using the check function $\rho_\tau$as defined by Koenker and Basette (1978) and for p=1, we have;

$$\min_{f\epsilon\mathfrak{F}} \sum_{i=1}^{n} \rho_\tau\big(y_i - f(x_i)\big) + \lambda\zeta(.) \qquad (1)$$

Given that $\zeta(.) = \frac{1}{2}\left(\int_0^1 (f''(x))^{\mathrm{q}}\right)^{1/\mathrm{q}}$

where $\mathfrak{F}$ is the functional class being focused on, $\zeta(\cdot)$ is a penalty on f to stop over fitting, and λ is a tuning parameter that checks the accuracy of fit and the smoothness by controlling the magnitude of $\zeta(\cdot)$and$\mathrm{q}$ is a positive integer. When $\mathrm{q} = 1$with an appropriate choice of $\mathfrak{F}$, Koenker et al. (1994) showed that the solution to equation (1) is a linear spline with knots at x_i; i = 1,…, n giving rise to a l_1 loss + l_1 penalty problem. Given $\mathrm{q} = 2$ equation (1) then becomes;

$$\min_{f\epsilon\mathfrak{F}} \sum_{i=1}^{n} \rho_\tau\big(y_i - f(x_i)\big) + \frac{\lambda}{2}\int_0^1 f''(x)^2\, dx \qquad (2)$$

As given by Bloomfield and Steiger (1983), but Li Y., Liu Y and Zhu J. (2007) came up with a more general approach that has a regular squared norm penalty and this is given by;

$$\min_{f\epsilon\mathfrak{H}_k} \sum_{i=1}^{n} \rho_\tau\big(y_i - f(x_i)\big) + \frac{\lambda}{2} \parallel f^2 \parallel_{\mathfrak{H}_k} \qquad (3)$$

Where $\mathfrak{H}_k$is an organized reproducing Kernel Herbert Space (RKHS) created by a positive definite Kernel $K(X, X')$. Using the representer's theorem the solution to equation (3) becomes;

$$\breve{f}(x) = \breve{\beta}_0 + \frac{1}{\lambda}\sum_{i=1}^{n} \breve{\theta}_i K(X_i, X) \qquad (4)$$

Where $K(X_i, .)$ is the ith kernel function for the training sample and $\breve{\theta} = (\breve{\theta}_1, \ldots \breve{\theta}_n)'$ is the estimated kernel function coefficient vector.Equation (4) can be rewritten giving us a kernel quantile regression thus;

$$\min_{\beta_0,\theta} \sum_{i=1}^{n} \rho_\tau \left(y_i - \beta_0 - \frac{1}{\lambda}\sum_{i=1}^{n} \theta_i K(X, X_i) + \frac{1}{2\lambda}\sum_{i=1}^{n}\sum_{\ddot{\imath}}^{n} \theta_i\theta_{\ddot{\imath}} K(X_i, X_{\ddot{\imath}})\right) \qquad (5)$$

There are many choices of the kernel function $K(.,.)$, but for this study we will focus on two radial basis Kernel. General Radial basis kernel is given as;

$$K(X, X') = \exp(- \parallel X, X' \parallel^2 / 2\sigma^2) \qquad (6)$$

Where sigma (σ) is pre-specified adjustable parameter, it is must important in the performance of the kernel.

The Gaussian kernel: This is a radial basis function and it is mainly used when there is no further prior knowledge of the data and it presented thus;

$$K(X, X') = exp\left(-\frac{\|X - X'\|^2}{2\sigma^2}\right) \quad (7)$$

The Laplace kernel is equally a radial basis function kernel but it less sensitive to the perturbation of the adjustable parameter sigma and it is given by;

$$K(X, X') = exp\left(-\frac{\|X - X'\|}{\sigma}\right) \quad (8)$$

Root Mean Square Error of Prediction (RMSEP): This is the sample standard deviation of the difference between observed and predicted and is given by Mulyatia et al (2018) as;

$$RMSEP = \sqrt{\frac{1}{n}\sum(y - \breve{y})^2} \quad (9)$$

Where y is the observed and $\breve{y}$ is the predicted.

Relative Absolute Error (RAE): RAE takes the total absolute error and normalizes it by dividing by the predicted. It isused as a measure of precision and it is given by;

$$RAE = \left|\frac{y - \breve{y}}{\breve{y}}\right| \text{X } 100 \quad (10)$$

Coefficient (r): This is a measure of the co linearity between the observed (y) and predicted ($\breve{y}$) values. It's value ranges from -1 to +1 and the value close to +1represents best fit. It is oversensitive to extreme value (Kumar et al., 2020; Tarate et al., 2021).

$$r = \left[\frac{\sum_{i=1}^{n}\{(y - \bar{y})(\breve{y} - \bar{\breve{y}})'\}}{\sqrt{\sum(y - \bar{y})^2}\sqrt{\sum(\breve{y} - \bar{\breve{y}})^2}}\right] \quad (11)$$

3. RESULTS

Table 1: Assessment of linear model assumptions

	Value	P-value	remark
Global stat	1.27e+05	0.000	Non-linear relationship
Skewness	3.519e+03	0.000	Non-normal
Kurtosis	1.24e+05	0.000	Non-normal
Link function	2.513e+00	0.1129	Dependent variable is continuous
Heteroscedasticity	1.042e+02	0.000	Error variance is not constant

Table 1 above reveals that there is a non-linear relationship between the response variable and each of the explanatory variables. The skewness and the kurtosis value reveal non-normal residuals, the link function tells us that the response variable is continuous and test for error variance reveals presence of heteroscedasticity.

Table 2: Predictions of first 30 values using Gaussian Kernel for different values of sigma for quantiles; 0.25, 0.5, 0.75 and 0.95

	Observed	**SIGMA VALUES**															
		5				**10**				**100**				**1000**			
		Tau				**Tau**				**Tau**				**tau**			
S/N		**0.25**	**0.5**	**0.75**	**0.95**	**0.25**	**0.5**	**0.75**	**0.95**	**0.25**	**0.5**	**0.75**	**0.95**	**0.25**	**0.5**	**0.75**	**0.95**
1	14.4	14.4	13.7	14.4	15.1	13.1	13.7	14.4	14.9	13.4	13.7	14.4	14.4	14.4	14.4	14.4	14.4
2	15.6	15.6	14.5	15.2	15.6	14.0	14.5	15.5	15.6	14.3	15	15.6	15.6	15.6	15.6	15.6	15.6
3	14.8	14.8	14.7	14.8	15.4	13.9	13.2	14.8	15.4	13.5	13.8	14.8	14.8	14.8	14.8	14.8	14.8
4	15	15	14.9	15	15	14.5	14.9	15	15	14.9	15	15	15	14.9	14.9	14.9	15
5	12.9	14.2	13.7	14.3	15	12.9	13.7	14.3	14.8	12.9	12.9	14.1	14.4	12.9	12.9	12.9	14.1
6	14.1	14.3	14.1	14.8	15.4	13.7	14.1	15.0	15.3	14.1	14.1	14.1	14.7	14.1	14.1	14.1	14.2
7	13	14.5	14.2	14.9	15.5	13.9	14.2	15.1	15.5	13	13.1	14.8	15.2	13	13	13	14.2
8	13.7	14.7	13.8	14.6	15.4	13.7	13.8	14.7	15.4	13.7	13.7	13.9	15.2	13.7	13.7	13.7	14.3
9	13.6	14.1	13.4	14.2	15.1	13.6	13.6	13.9	15.1	13.6	13.6	13.7	14.4	13.6	13.6	13.6	14.2
10	13.4	13.9	13.4	13.9	14.8	13.4	13.4	13.5	14.7	13.4	13.4	13.4	14.1	13.4	13.4	13.4	14.2
11	12.8	13.9	12.8	13.2	14.1	12.8	11.8	12.8	14.1	12.8	12.8	12.8	14.2	12.8	12.8	12.8	14.2
12	11.8	13.9	11.8	11.8	12.9	11.8	11.8	11.8	13.6	11.8	11.8	11.8	14.3	11.8	11.8	11.8	14.2
13	12.2	13.4	12.2	13.2	12.8	11.4	12.2	12.2	12.7	12.2	12.2	12.2	14.1	12.2	12.2	12.2	14.2
14	11.1	11.4	10.8	11.7	12.8	10.2	10.9	11.6	12.6	10.5	10.4	11.1	11.8	11.1	11.1	11.1	13.5
15	12.8	12.8	9.9	11.2	12.8	8.82	9.06	11.3	12.8	9.5	11.1	12.8	12.8	12.8	12.8	12.8	14.1
16	11.3	13.5	12.2	12	13.3	11.3	11.4	11.3	13.2	11.3	11.3	11.3	13.9	11.3	11.3	11.3	14.2
17	12.4	12.4	9.9	11.6	12.7	9.43	10.2	11.9	12.5	10.7	11.7	10.9	12.4	12.4	12.4	12.4	13.8
18	10.	11.	10.	11.	12.	9.9	10.	11.	12.	10.	10.	10.	11.	10.	10.	10.	13.

	2	1	6	6	8	5	4	5	5	2	2	9	5	2	2	2	3
19	9.4	12.9	10.9	11.7	12.9	9.4	9.98	11.9	13.0	9.4	9.4	11.9	13.7	9.4	9.4	9.4	14.2
20	9.3	11.1	10.2	11.4	12.8	9.44	9.87	11.2	12.6	9.3	9.7	11.2	11.3	9.3	9.3	9.3	13.7
21	10.3	12.6	11.1	12.9	12.8	10.3	11.7	13.1	12.8	10.3	10.6	12	13	10.3	10.3	10.3	13.9
22	10.5	11.7	10.5	11.7	12.7	10.1	10.5	11.8	12.5	10.5	10.5	11.3	12.1	10.5	10.5	10.5	13.9
23	10.5	11.2	10.2	11.5	12.7	9.33	10.1	11.4	12.6	10.1	10.5	10.5	11.5	10.5	10.5	10.5	13.8
24	10.3	10.7	7.9	10.3	12.5	7.69	8.04	10.3	12.5	8.2	9.2	10.3	11.3	10	10.3	10.3	12.7
25	12.6	12.6	8.9	11.1	12.6	8.47	9.36	11.4	12.6	9.3	11.2	11.9	12.6	12.9	12.6	12.6	12.7
26	11.9	12.7	11.9	12.1	13.1	11.3	11.9	12.1	12.9	11.9	12.1	12.1	13.2	11.9	11.9	11.9	13.4
27	12.1	12.7	11.9	12.1	13.1	11.4	11.9	12.1	12.9	11.9	12.1	12.1	13.2	12.1	12.1	12.1	13.4
28	12.9	12.9	10.8	11.7	12.9	9.88	10.6	12.2	12.9	12.9	12.9	12.9	13.6	12.9	12.9	12.9	14
29	12.7	12.7	10.2	12.5	12.7	9.57	10.5	11.8	12.7	10.9	12.7	12.7	12.7	12.7	12.7	12.7	14
30	12.9	13.7	11.9	12.9	12.9	11	12.4	12.9	12.9	12.9	12.9	12.9	14.1	12.9	12.9	12.9	14.2

Table 2 above presents the prediction values at all quantiles considered using the Gaussian kernel.

Table 3: Prediction Error associated with the predictions for Gaussian kernel

Sigma values	Quantiles			
	0.25	0.50	0.75	0.95
5	1.664	1.348	1.117	1.683
10	1.194	1.376	1.095	1.633
100	1.114	0.547	0.844	1.536
1000	0.077	0.000	0.000	2.125

From the table 3 above we can see that the prediction error is approximately 0 from sigma =1000, except for the 95th quantile. The prediction error of the 95th quantile is seen to be on the increase as the sigma value increases while for the other quantiles, the error decreases with increased sigma value. Hence Gaussian kernel predicts well when sigma is greater or equal to 1000.

Table 4: Relative Absolute Error (RAE) associated with the predictions for Gaussian kernel

Sigma values	RAE for Quantiles			
	0.25	0.50	0.75	0.95
5	192.5%	257%	209%	294%
10	311%	251%	178%	282%
100	166%	78.2%	109%	258%
1000	5.9%	0.6%	0.6%	360%

Table 4 above shows the RAE at all quantiles, and it can be seen that the least value is at the 50th and 75th quantile for sigma value of 1000. Hence showing that for Gaussian kernel the best sigma value is 1000 and above.

Table 5: Correlations between observed and predicted value for each quantile with Gaussian kernel

Sigma values	Correlation for Quantiles			
	0.25	0.50	0.75	0.95
5	0.849	0.731	0.782	0.767
10	N/A	N/A	N/A	N/A
100	0.841	0.950	0.871	0.752
1000	0.998	0.999	0.999	0.642

Table 5 displays the correlation between the observed and the estimated values at all quantiles. The results show a very high correlation at all sigma values at all quantiles. But at sigma equals to 1000 the correlation coefficient is very high at all quantiles except at the 90th quantile. It is seen that the correlation coefficient reduces as the sigma increases at the 95th quantile.

Table 6: Paired T-test between Observed and predictions from each quantile for Gaussian kernel

Sigma values	P-values			
	τ =0.25	τ =0.50	τ =0.75	τ =0.95
5	0.0000	0.0667	0.021	0.0000
10	0.0002	0.0225	0.036	0.0000
100	0.0033	0.0200	0.050	0.0001
1000	0.8299	0.326	0.326	0.0000

From the paired t-test in table 6, the mean difference between the observed and the predicted values are significantly different at all sigma values except for sigma = 1000. But 95th quantile p-value for sigma=1000 differs. We can say that for accurate predictions using the Gaussian kernel, the sigma value must be very high at least 1000 or above. Also we can note that prediction of the heavy tail comes with high prediction error, hence producing inaccurate predictions.

Table 7: Predictions of first 30 values using Laplace Kernel for different values of sigma for quantiles; 0.25, 0.5, 0.75 and 0.95

		SIGMA VALUES															
		5				**10**				**100**				**1000**			
	Observe	**Tau**				**Tau**				**Tau**				**Tau**			
S/N		**0.25**	**0.5**	**0.75**	**0.95**	**0.25**	**0.5**	**0.75**	**0.95**	**0.25**	**0.5**	**0.75**	**0.95**	**0.25**	**0.5**	**0.75**	**0.95**
1	14.4	13.9	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4	14.4
2	15.6	14.7	15.6	15.6	15.6	15.7	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6
3	14.8	14.4	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8
4	15	14.9	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
5	12.9	12.9	12.9	13.2	14.2	12.9	12.9	12.9	14.1	12.9	12.9	12.9	14.3	12.9	12.9	12.9	14.3
6	14.1	14.1	14.1	14.1	14.6	14.1	14.1	14.1	14.3	14.1	14.1	14.1	14.3	14.1	14.1	14.1	14.3
7	13	13	13	13.5	14.8	13	13	13	14.5	13	13	13	14.3	13	13	13	14.3
8	13.7	13.7	13.7	13.7	14.9	13.7	13.7	13.7	14.7	13.7	13.6	13.7	14.3	13.7	13.7	13.7	14.3
9	13.6	13.6	13.6	13.6	14.3	13.6	13.6	13.6	14.1	13.6	13.6	13.6	14.3	13.6	13.6	13.6	14.3
10	13.4	13.4	13.4	13.4	14.1	13.4	13.4	13.4	13.9	13.4	13.4	13.5	14.3	13.4	13.4	13.4	14.3
11	12.8	12.8	12.8	12.8	13.7	12.8	12.8	12.8	13.9	12.8	12.8	12.8	14.3	12.8	12.8	12.8	14.3
12	11.8	11.8	11.8	11.8	13.3	11.8	11.8	11.8	13.9	11.8	11.8	11.8	14.3	11.8	11.8	11.8	14.3
13	12.2	12.2	12.2	12.2	12.6	12.2	12.2	12.2	13.4	12.2	12.2	12.2	14.3	12.2	12.2	12.2	14.3
14	11.1	10.9	11.1	11.1	11.5	11.1	11.1	11.1	11.4	11.1	11.1	11.1	14.2	11.1	11.1	11.1	14.3
15	12.8	10.2	11.5	12.8	12.8	11.6	12.8	12.8	12.8	12.8	12.8	12.8	14.3	12.8	12.8	12.8	14.3
16	11.3	11.3	11.3	11.3	12.9	11.3	11.3	11.3	13.5	11.3	11.3	11.3	14.3	11.3	11.3	11.3	14.3
17	12.4	11.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	14.3	12.4	12.4	12.4	14.3
18	10.2	10.2	10.2	10.4	11.4	10.2	10.2	10.2	11.1	10.2	10.2	10.2	14.2	10.2	10.2	10.2	14.3
19	9.4	9.4	9.4	11.1	12.7	9.4	9.4	9.9	12.8	9.4	9.4	9.4	14.3	9.4	9.4	9.4	14.3

20	9.3	9.3	9.3	9.7	11.2	9.3	9.3	9.3	11	9.3	9.3	9.3	14.3	9.3	9.3	9.3	14.3
21	10.3	10.3	10.3	11.4	12.6	10.3	10.3	10.5	12.6	10.3	10.3	10.3	14.3	10.3	10.3	10.3	14.3
22	10.5	10.6	10.5	10.5	11.7	10.5	10.5	10.5	11.7	10.5	10.5	10.5	14.3	10.5	10.5	10.5	14.3
23	10.5	10.5	10.5	10.5	11.3	10.5	10.5	10.5	11.2	10.5	10.5	10.5	14.3	10.5	10.5	10.5	14.3
24	10.3	8.4	8.9	10.3	11.2	9.03	9.9	10.3	10.7	10.3	10.3	10.3	13.9	10.3	10.3	10.3	14.3
25	12.6	9.4	11.2	12.4	12.6	10.4	12.1	12.6	12.6	12.6	12.6	12.6	14.1	12.6	12.6	12.6	14.3
26	11.9	11.9	11.9	11.9	12.4	11.9	11.9	11.9	12.7	11.9	11.9	11.9	13.8	11.9	11.9	11.9	14.3
27	12.1	12.1	12.1	12.1	12.4	12.1	12.1	12.1	12.7	12.1	12.1	12.1	13.9	12.1	12.1	12.1	14.3
28	12.9	12.3	12.9	12.9	12.9	12.9	12.9	12.7	12.9	12.9	12.9	12.9	14.4	12.9	12.9	12.9	14.3
29	12.7	11.3	12.7	12.7	12.7	12.6	12.7	12.7	12.7	12.7	12.7	12.7	14.3	12.7	12.7	12.7	14.3
30	12.9	12.9	12	12.9	12.9	12.9	12.9	12.9	13.6	12.9	12.9	12.9	14.3	12.9	12.9	12.9	14.3

Table 8: Prediction Error associated with the predictions for Laplace kernel

Sigma values	Quantiles			
	0.25	0.50	0.75	0.95
5	0.917	0.463	0.395	0.351
10	0.517	0,117	0.105	1.174
100	0.000	0.012	0.012	2.459
1000	0.000	0.000	0.000	2.516

From table 8 above it can be seen that the prediction error is approximately 0at sigma equals or greater than10, except for the 95th quantile. The prediction error of the 95th quantile is seen to also be on the increase as the sigma value increases while for the other quantiles, the error decreases with increased sigma.

Table 9: Relative Absolute Error associated with the predictions for Laplace kernel

Sigma values	RAE for Quantiles			
	0.25	0.50	0.75	0.95
5	124%	47%	38.6%	183%
10	46.9%	8%	8.5.%	189%
100	0%	0.7%	0.7%	418%
1000	0%	0%	0%	427%

Table 9 above shows the RAE at all quantiles, and it can be seen at 25th, 50th and 75th quantile the RAE is approximately 0 or 1for sigma values of 100 and 1000. Hence revealing that Laplace kernel function produce very good predictions at sigma values 100 and above.

Table 10: Correlations between observed and predicted value for each quantile with Laplace kernel

Sigma values	Correlation for Quantiles			
	0.25	0.50	0.75	0.95
5	0.884	0.964	0.975	0.869
10	0.96	0.99	0.99	0.86
100	1	0.99	0.99	0.58
1000	1	1	1	0.56

Table 10 reveals an almost perfection correlation between the observed and the estimated at all quantiles for all sigma values. It is worthy of note that as the sigma values increase the correlation coefficient reduces at the 95th quantile.

Table 11: Paired T-test between Observed and predictions from each quantile for Laplace kernel

Sigma values	P-values			
	τ =0.25	τ =0.50	τ =0.75	τ =0.95
5	0.008	0.046	0.063	0.000
10	0.097	0.164	0.393	0.000
100	0.326	0.326	0.326	0.000
1000	N/A	N/A	N/A	N/A

From the paired t-test in table 7, the mean difference between the observed and the predicted values are not significantly different at all sigma values except for sigma = 5, for 25th and 50th quantiles. We can say that Laplace produces accurate predictions for sigma greater than 5 at all quantiles.

4. DISCUSSIONS

The paper presented Kernel quantile regression in prediction of inflation rate based on foreign exchange, crude oil price and currency in circulation. Two kernel functions namely the Gaussian kernel and the Laplace kernel were applied using four different sigma values in the analysis. Predictions at four quantiles (0.25, 0.50, 0.75 and 0.95) were made and comparison of the predicted values and the observed values were done based on the Root Mean Square Error of Prediction, Relative absolute error, Correlation and Paired T-test. The result in table 2 and table 7 shows the first 30 predicted values at all quantiles for the Gaussian kernel and Laplace kernel respectively. Non linearity test, normality test, Heteroscedasticity test were carried out and table 1 presented the result which reveals non-linearity, non-normality and presence of heteroscedasticity. Also the result in table 1 shows that the response variable is a continuous variable. The table 3 outlines the prediction errors for the Gaussian kernel and it can see that the prediction error is approximately 0 from sigma =1000, except for the 95th quantile. The prediction error of the 95th quantile is seen to be on the increase as the sigma value increases while for the other quantiles, the error decreases with increased sigma value. Hence Gaussian kernel predicts well when sigma is greater or equal to 1000. Table 4 outlines the results of RAE at all quantiles, and it is observed that the least value is at the 50th and 75th quantile for sigma value of 1000. The correlation result in table 5 shows very high correlation but it was noticed that for the 90th quantile the correlation coefficient reduces as the sigma value increases. The paired t-test in table 6, showed that the mean difference between the observed and the predicted values are significantly different at all sigma values except for sigma = 1000. But 95th quantile p-value for sigma=1000 differs. Also it was noted that prediction of the heavy tail comes with high prediction error, hence producing inaccurate predictions. Based on this it can be concluded that the Gaussian Kernel is very sensitive to the sigma value because the lower the sigma the higher the prediction error which is seen to reduce as the sigma increases but produces predictions that are highly correlated to the observed values. The best predictions with Gaussian kernel are seen to be at sigma value of 1000 and above.

Results of the Laplace kernel as shown in table 8 show that the prediction error is approximately 0 at sigma value 10 or greater than10, except for the 95th quantile. The prediction error of the 95th quantile is observed to also be on the increase as the sigma value increases while for the other quantiles, the error decreases with increased sigma. Table 9 shows the RAE at all quantiles, and it can be seen at 25th, 50th and 75th quantile the RAE is approximately 0 or 1for sigma values of 100 and 1000. The correlation coefficient values in table 10 presents an almost perfection correlation between the observed and the estimated at all quantiles for all sigma values. It is worthy of note that as the sigma values increase the correlation coefficient reduces at the 95th quantile. These results show that the Laplace kernel is less sensitive to the sigma as it produces better prediction at almost the whole considered sigma values. This study has been able to show that Gaussian kernel is very sensitive to the sigma value, therefore researchers are advised to use sigma values1000 or above in other to get very good predictions when using Gaussian kernel. But when using Laplace kernel sigma value above 10 will produce very good predictions.

REFERENCE

[1] Bang S., Soo-Heabg E., Jhun M and Cho H. J. (2017). Composite kernel quantile regression. Communication in Statistics - Simulation and Computation; 46(3), pp 2228-2240.

[2] Cattaneo M. D.,Jansson M. and Ma X (2019). Simple Local Polynomial Density Estimators. Journal of the American Statistical Association; 115(531):1-11.

[3] Hastie, T., Tibshirani, R., Friedman, J. (2009). Kernel Smoothing Methods. In: The Elements of Statistical Learning. Springer Series in Statistics. Springer, New York, NY; pp 191-218. https://doi.org/10.1007/978-0-387-84858-7_6

[4] Huber P. J. (1981) Robust Statistics. John Wiley and Sons, New York.

[5] Koenker R. and Basette G. Jr, (1978). Regression Quantiles. Econometrica; 46(1), pp 33-50.

[6] Koenker R. and Hallock K. F. (2001). Journal of Economic Perspectives; 15(4), pp 143–156.

[7] Koenker R.(2005). Quantile Regression. Cambridge University Press.

[8] Kumar P., Sharma N. and Rana A., (2012). Article: Handwritten Character Recognition using Different Kernel based SVM Classifier and MLP Neural Network (A COMPARISON). International Journal of Computer Applications 53(11):25-31, September 2012.

[9] Li Y., Liu Y and Ji Zhu J., (2007). Quantile Regression in Reproducing Kernel Hilbert Spaces, Journal of the American Statistical Association, 102:477, 255-268, DOI: 10.1198/016214506000000979

[10] Lee E. R., Cho J. and Park S. (2022). Penalized kernel quantile regression for varying coefficient models. Journal of Statistical Planning and Inference; 217 Pp 8-23. https://doi.org/10.1016/j.jspi.2021.07.003

[11] Mulyatia A. E. ,Wigenab A. H., and Djuraidah A. (2018). Statistical Downscaling Using Kernel Quantile Regression to Predict Extreme Rainfall. International Journal of Sciences: Basic and Applied Research; 42(2), pp 1-9.

[12] Scholkopf, A. J. Smola, R. C. Williamson, and P. L. Bartlett (2000). New support vector algorithms. Neural Computation, 12:1207–1245.

[13] Takeuchi I., V.le Q., Sear t. and Smola A. F. (2006). Non-Parametric Regresion. Journal of Machine Learning Research; 7, pp1231-1264.

[14] Vinod H. J. (1978). A survey of ridge regression and related techniques for improvements over ordinary least squares. Review of Economics and Statistics, 60:121–131.

]15] Wang, X., Smith-Miles, K., and Hyndman, R. (2009). Rule induction for forecasting method selection: meta-learning the characteristics of univariate time series. *Neurocomputing* 72, 2581– 2594.doi: 10.1016/j.neucom.2008.10.017